Odds (modified from Crawley 2007, the R book)

The trick of linearizing the logistic model involves a very simple transformation:

Bookmarkers specify probabilities by quoting the odds against a particular horse winning a race.

Thus, where the scientist might state a proportion as 0.667 (2 out of 3) the bookmarker will give odds of 2 to 1. In symbols: p= probability, odds= p/q

Now if we take the odds p/q and substitute this into the formula for the logistic model, we get:

$$\frac{p}{q} = \frac{e^{a+bx}}{1+e^{a+bx}}\left[1-\frac{e^{a+bx}}{1+e^{a+bx}}\right]^{-1}$$

That can be simplified to (try yourself)

$$\frac{p}{q} = \frac{e^{a+bx}}{1+e^{a+bx}}\left[1-\frac{e^{a+bx}}{1+e^{a+bx}}\right]^{-1} = e^{a+bx}$$

Taking the natural log it simplifies to:

$$\ln\left(\frac{p}{q}\right) = a + bx$$

This gives the linear predictor, a + bx, not for p but for the logit transformation, namely ln(p/q).