Useful properties of the normal distribution

The normal distribution has useful properties:
Can be added:

$$E(X+Y) = E(X) + E(Y)$$

$$\sigma^2(X+Y) = \sigma^2(X) + \sigma^2(Y)$$

Can be transformed with **shift** and **change of scale** operations

Consider two random variables X and Y:

Let X~N(μ,σ) and let Y=aX+b where a and b are constants

Change of scale is the operation of multiplying X by a constant a because one unit of X becomes "a" units of Y.

Shift is the operation of adding a constant b to X because we simply move our random variable X "b" units along the x-axis.

If X is a normal random variable, then the new random variable Y created by these operations on X is also a normal random variable.

For X~N(μ,σ) and Y=aX+b

$$E(Y) = a\mu + b$$

$$\sigma^2(Y) = a^2\sigma^2$$

A special case of a change of scale and shift operation in which a = 1/σ and b = -1(μ/σ):
Y = (1/σ)X-(μ/σ) = (X-μ)/σ

This gives E(Y)=0 and σ^2(Y)=1

Thus, any normal random variable can be transformed to a standard normal random variable.