



P.E.M.D.A.S. Order of Operations

February 10, 2018

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Solve an Expression

What is the following expression equal to?

$$6 + 2 \times 2$$

16 or 10

Motivation for "PEMDAS"

An expression should have a definite answer - we all should AGREE whether $6 + 2 \times 2$ is 16 or 10.

- ▶ Parenthesis
- ▶ Exponents
- ▶ Multiplication and division
- ▶ Addition and subtraction

Now what is the answer? 16 or 10

HOT POTATO!

If you are tossed the ball - you say the next order of operation.
Then you toss the ball to another student.

▶ $2 \times (3 + 4) + 30 - 1$

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▶ $2 \times (3 + 4) + 30 - 1$

▶ $2 \times 7 + 30 - 1$

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- ▶ $2 \times (3 + 4) + 30 - 1$
- ▶ $2 \times 7 + 30 - 1$
- ▶ $14 + 30 - 1$

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- ▶ $2 \times (3 + 4) + 30 - 1$
- ▶ $2 \times 7 + 30 - 1$
- ▶ $14 + 30 - 1$
- ▶ $44 - 1$

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- ▶ $2 \times (3 + 4) + 30 - 1$
- ▶ $2 \times 7 + 30 - 1$
- ▶ $14 + 30 - 1$
- ▶ $44 - 1$
- ▶ 43

HOT POTATO

► $(2 + 36 \div 12) + (1 + 3)^2$

HOT POTATO

- ▶ $(2 + 36 \div 12) + (1 + 3)^2$
- ▶ $(2 + 36 \div 12) + (4)^2$

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- ▶ $(2 + 36 \div 12) + 16$

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- ▶ $(2 + 36 \div 12) + (1 + 3)^2$
- ▶ $(2 + 36 \div 12) + (4)^2$
- ▶ $(2 + 36 \div 12) + 16$
- ▶ $(2 + 3) + 16$

HOT POTATO

- ▶ $(2 + 36 \div 12) + (1 + 3)^2$
- ▶ $(2 + 36 \div 12) + (4)^2$
- ▶ $(2 + 36 \div 12) + 16$
- ▶ $(2 + 3) + 16$
- ▶ $5 + 16$

HOT POTATO

- ▶ $(2 + 36 \div 12) + (1 + 3)^2$
- ▶ $(2 + 36 \div 12) + (4)^2$
- ▶ $(2 + 36 \div 12) + 16$
- ▶ $(2 + 3) + 16$
- ▶ $5 + 16$
- ▶ 21

Progress POLL

I feel that I need some more practice with PEMDAS. Can you give another example?

I feel confident with PEMDAS. Let's move on!

Are You Ready For a Little Competition?

The first person to hold up a piece of paper with the correct answer on it gets **FIRST PLACE**. Shouting out the answer will disqualify you for this round. You only get one chance to hold up the right answer. Chris and I will be the judge of who raised their paper first - a tie will be broken with a coin flip. We will have three rounds!

Round 1: The Battle for FIRST PLACE

Are you ready?

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Are you ready?
On your mark...

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Are you ready?
On your mark...

$$(3 + 3)^2 \div 12 \div 1 + 2^2$$

Round 1: The Battle for FIRST PLACE

Are you ready?
On your mark...

$$(3 + 3)^2 \div 12 \div 1 + 2^2$$

The correct answer was 7. Would the winner please write the solution on the board and explain to the class?

Round 2: The Battle for SILVER

Are you ready?

Round 2: The Battle for SILVER

Are you ready?
On your mark...

Round 2: The Battle for SILVER

Are you ready?
On your mark...

$$5 \times 2^2 + 2 \times 5 + 2$$

Round 2: The Battle for SILVER

Are you ready?
On your mark...

$$5 \times 2^2 + 2 \times 5 + 2$$

The correct answer was 32. Would the winner please write the solution on the board and explain to the class?

Round 3: The Battle for BRONZE

Are you ready?

Round 3: The Battle for BRONZE

Are you ready?
On your mark...

Round 3: The Battle for BRONZE

Are you ready?
On your mark...

$$4 \div (2 \times 3 - 2) - 1$$

Round 3: The Battle for BRONZE

Are you ready?
On your mark...

$$4 \div (2 \times 3 - 2) - 1$$

The correct answer was 0. Would the winner please write the solution on the board and explain to the class?

BEAT THE TEACHER

Who would the class like to challenge?

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Ms. Hanna

Mr. Chris

BEAT THE TEACHER

Who would the class like to challenge?

Ms. Hanna

Mr. Chris

(Whoever is not being challenged will write the problem on the board)

Don't be so NEGATIVE!!

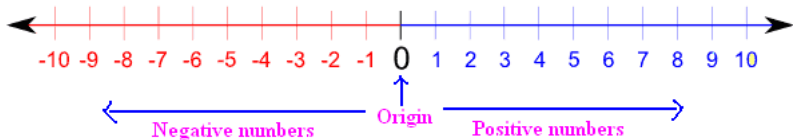
First, a poll: I am comfortable with negative numbers.
I'm not really comfortable with negative numbers.

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I'm not really comfortable with negative numbers.

What is a negative number?

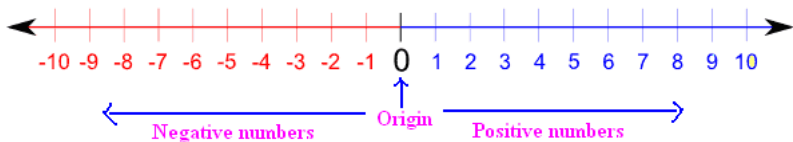


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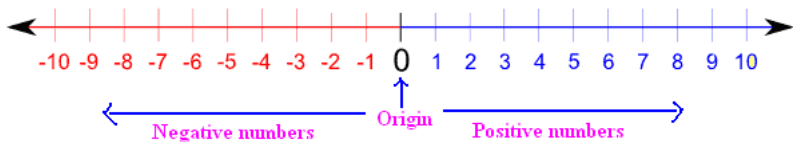
Making a number negative is just like reflecting it over 0. For example, 3 and -3 are the same distance away from 0.

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First, a poll: I am comfortable with negative numbers.

I'm not really comfortable with negative numbers.

What is a negative number?



Making a number negative is just like reflecting it over 0. For example, 3 and -3 are the same distance away from 0.

Did you know that subtracting a number is the same thing as adding a negative?

Multiplying and Dividing Negatives

hold up RED if the answer is NEGATIVE

hold up BLUE if the answer is POSITIVE

▶ $-18 \div 3$

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▶ $-18 \div 3 = -6$

▶ $(-1) \times (-2)$

Multiplying and Dividing Negatives

hold up RED if the answer is NEGATIVE

hold up BLUE if the answer is POSITIVE

▶ $-18 \div 3 = -6$

▶ $(-1) \times (-2) = 2$

▶ $(-4)^2$

Multiplying and Dividing Negatives

hold up RED if the answer is NEGATIVE

hold up BLUE if the answer is POSITIVE

▶ $-18 \div 3 = -6$

▶ $(-1) \times (-2) = 2$

▶ $(-4)^2 = 16$

▶ $3 \times 4 \times (-3)$

Multiplying and Dividing Negatives

hold up RED if the answer is NEGATIVE

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▶ $-18 \div 3 = -6$

▶ $(-1) \times (-2) = 2$

▶ $(-4)^2 = 16$

▶ $3 \times 4 \times (-3) = -36$

▶ -2^2

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▶ $-18 \div 3 = -6$

▶ $(-1) \times (-2) = 2$

▶ $(-4)^2 = 16$

▶ $3 \times 4 \times (-3) = -36$

▶ $-2^2 = -4$ because the negative is NOT in the square!!

Review of Fractions

What do you think of fractions?

- ▶ I HAAATTTTEEEE fractions with a die hard passion!!
- ▶ (Just raise your hand) fractions are okay
- ▶ Fractions are the COOOOLEST numbers!

Does anyone want to elaborate on why they feel this way?

What are fractions?

Which is prettier $0.333333333\dots$ or $\frac{1}{3}$?

What are fractions?

Which is prettier $0.3333333333\dots$ or $\frac{1}{3}$?

What if I told you that fractions are nothing but division?

Try with your calculator:

$$1 \div 3 = \frac{1}{3}$$

Adding and Subtracting Fractions

How do we add fractions?

1. Get a common denominator!
2. Add across the top!

Example:

$$\begin{aligned}\frac{1}{3} + \frac{1}{10} \\ &= \frac{1}{3} \times \frac{10}{10} + \frac{1}{10} \times \frac{3}{3} \\ &= \frac{10}{30} + \frac{3}{30} = \frac{13}{30}\end{aligned}$$

Another Example

► $\frac{7}{9} - \frac{25}{27}$

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$$\blacktriangleright \frac{7}{9} - \frac{25}{27}$$

$$\blacktriangleright = \frac{7}{9} \times \frac{3}{3} - \frac{25}{27}$$

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$$\blacktriangleright = \frac{7}{9} \times \frac{3}{3} - \frac{25}{27}$$

$$\blacktriangleright = \frac{21}{27} - \frac{25}{27}$$

$$\blacktriangleright = -\frac{4}{27}$$

Multiplying Fractions

How to multiply two fraction:

1. Multiply across the numerator and divisor
2. Simplify by canceling out any common factors

Who is familiar with this process?

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Who is familiar with this process?

- ▶ I am ready to do some problems!
- ▶ I would like you to elaborate

Example

$$\frac{2}{3} \times \frac{33}{6}$$

Example

$$\frac{2}{3} \times \frac{33}{6}$$

$$= \frac{66}{18}$$

Always ask the question: can I simplify this?

$$= \frac{6 \times 11}{6 \times 3} = \frac{11}{3}$$

Another Example

$$\frac{5}{14} \times \frac{7}{10}$$

Another Example

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WAIITTTT! - is there a more efficient way to solve this problem?

Another Example

$$\frac{5}{14} \times \frac{7}{10}$$

WAIITTTT! - is there a more efficient way to solve this problem?

$$= \frac{5}{2 \times 7} \times \frac{7}{2 \times 5} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Another Example

$$\frac{5}{14} \times \frac{7}{10}$$

WAIITTTT! - is there a more efficient way to solve this problem?

$$= \frac{5}{2 \times 7} \times \frac{7}{2 \times 5} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Guarantee: if you cancel ALL common factors out BEFORE you multiply - you won't have to do it at the end

Dividing Fractions

How to divide fractions:

- ▶ Multiply the numerator by the reciprocal of the denominator
- ▶ Simplify if you can

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What is happening when you divide two numbers? (we've discussed it already!)

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Why?

What is happening when you divide two numbers? (we've discussed it already!)

$$2 \div 3 = 2 \times \frac{1}{3} = \frac{2}{3}$$

Dividing Example

Which is the numerator? Which is the denominator? How do we know which to "flip"?

$$\frac{2}{3} \div \frac{5}{21}$$

Dividing Example

Which is the numerator? Which is the denominator? How do we know which to "flip"?

$$\frac{2}{3} \div \frac{5}{21}$$

$$= \frac{2}{3} \times \frac{21}{5} = \frac{2}{\color{blue}3} \times \frac{\color{blue}3 \times 7}{5} = \frac{14}{5}$$

How is this class going?

What would you like from me to make this class better?

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More activities?

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More activities?

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More activities?

How is the pace?

Any special requests?