Name: _____ School Grade: _____

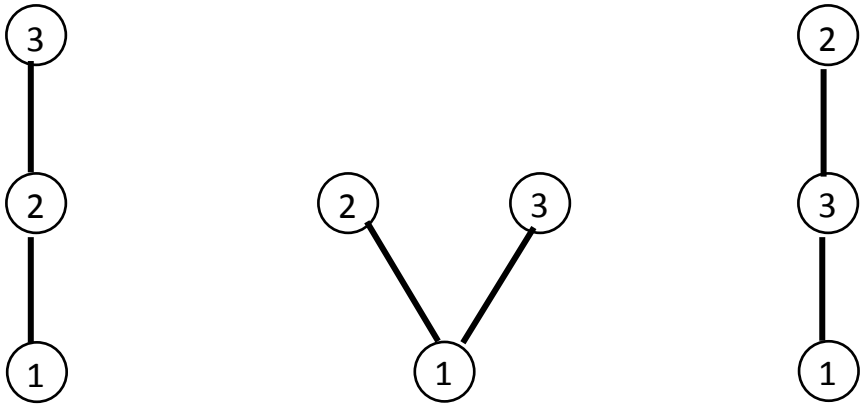
Central Florida Math Circle

University of Central Florida
Department of Mathematics

Some of the following problems might contain several parts, which are often of increasing difficulty levels. Please try your best to challenge yourself but do not worry if you cannot complete all of them. Good luck and have fun!

<u>Problem 1 (Counting Trees)</u> The following is a tree (connected but no cycle) of 2 nodes and there are no other trees of 2 nodes.

The number of trees containing 3 nodes is 3, as shown below.



(a) Can you draw all trees that contain 4 nodes? *The number might be larger than the one you first thought.*

Name: _____ School Grade: _____

(b) How many trees are there containing 5 nodes? *You might not want to draw all of them.*

(c) Can you provide and prove a formula for the number of trees containing n nodes, where n is a positive integer? *Proof is a kind of logic arguments to convince people why your formula is correct.*

Problem 2 (Exploring the Desert) A group of students organize an expedition to explore the interior of a vast desert region. They arrive at the edge of the desert with supplies (petrol, water, food, etc.) for a journey of 1600 miles. Unfortunately, they only have one lorry, and fully laden it can only carry sufficient supplies for 400 miles. What is the greatest distance they can travel into the desert and return safely? Explain how it works. *It is understood that supplies can be stashed in the desert for later use.*

Please submit your solutions to all or part of problems to ucfmathcircle@gmail.com by ~~October 5th~~ October 12th and we will announce the best solutions on our website on ~~October 6th~~ October 13th. Please feel free to use additional papers if necessary.