"Octopus! Come out and see my new socks!" he said.

"Silly Squid," said Octopus. "Don't you know we wear mittens?"

"No we don't. We wear socks," said Squid with a frown.

"Mittens!" said Octopus with a stomp.

"Socks!" said Squid with a double stomp.

MITTENS!
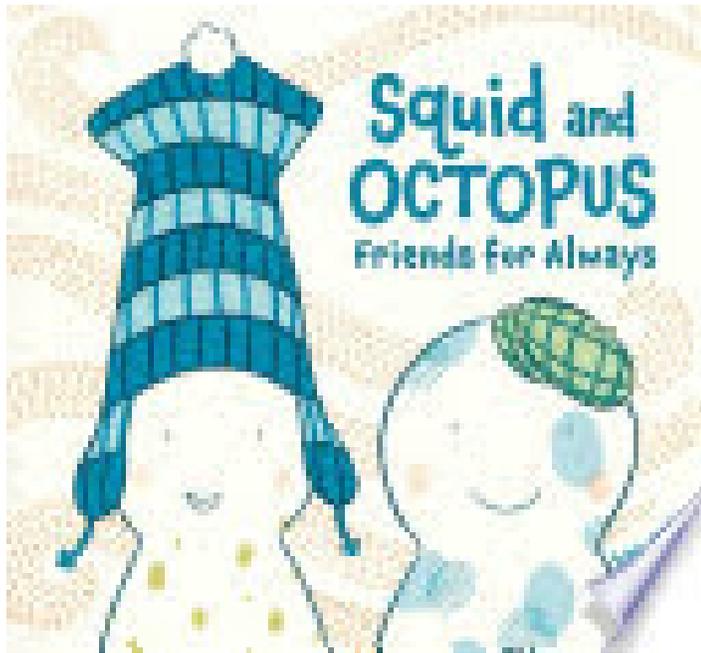
SOCKS!!

MITTENS!!

SOCKS!!

PIGEONHOLE PRINCIPLE

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Octopus: Socks or mittens ?

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"Mittens!" said Octopus with a stomp.
"Socks!" said Squid with a double stomp.

MITTENS!
Socks!
MITTENS!!
SOCKS!!

Octopus: Socks or mittens ?

This octopus wants to visit UCF with wearing socks. He has a drawer with socks: each sock is a color featured on the UCF Knights Logo. *Question: Without looking, how* many socks must he pick in order to have 8 socks of the same color?

Pigeonhole Principle

If 10 pigeons fly into 9 pigeonholes, is there one hole must contain two or more pigeons?

9 holes, and 10 = 9 + 1 pigeons.

So at least 1 hole contains at least 2 pigeons.

Pigeonhole Principle

If you place *n* + 1 objects in *n* holes, then at least one hole must contain more than one object.

The worst case:

Let us **suppose** that each hole contains less than 2 objects; then the total number of objects would be less than n, contradiction.

WOW! That is so easy! ☺

Same Birthday?

There are about 32 students in the Math Circle.

At least two students have the same day for their birthdays !! (regardless months)

What are the pigeonholes?

What are the pigeons?

What do we know?

What can we think of?

There are about 32 students in the Math Circle. At least two students have the same day for their birthdays !! (regardless months)

The pigeonholes are the days of any month.

The pigeonholes are then as follows
- 1st day of a month
- 2nd day of a month
- 3rd day of a month
- ...
- 31st day of a month

So, there are 31 pigeonholes!

There are 32 students.

So, it's true!

- What do we know ... about knowing people?
- It's a pigeonhole problem ...
  - So what are the pigeonholes?
  - What are the pigeons?

Same number of Friends?

You are in a room with 10 other people.

At least two people have the same number of friends (know each other) in the room.

Is this true ?

What do we know … about knowing people?
It's a pigeonhole problem …
- So what are the pigeonholes?
- What are the pigeons?

You are in a room with 10 other people. *There are at least two people in the room have the same number of friends in the room*

If Anne is Rachel's friend, then Rachel is Anne's friend.

The pigeonholes are the number of friends in the room.

The pigeonholes are then as follows
- 0 friend for somebody
- 1 friend for somebody
- 2 friends for somebody
- …
- 10 friends for somebody

But wait! It is not possible to have someone who has ten friends
at the same time as there is someone in the room that knows nobody!

So, there can only be 10 pigeonholes!

There are 11 people

So, it's true!

- What do we know … about knowing people?
- It's a pigeonhole problem …
    - So what are the pigeonholes?
    - What are the pigeons?

Same Grades?

Nine 5th graders have FSA tests for Math, ELA and Sciences which is given grade S or F.

At least two students get the same grades for three tests.

(example: S for Math & ELA; F for Science)

Is this true?

What do we know … about grades for 3 tests?

It's a pigeonhole problem …

- So what are the pigeonholes?
- What are the pigeons?

Nine 5th graders have FSA tests for Math, ELA and Science. The possible grades are: S or F. At least two students get the same grades for three tests.

The pigeonholes are all possible grades for 3 tests.

The pigeonholes are then as follows
- Math: S, ELA: S, Science: S
- Math: S, ELA: S, Science: F
- Math: S, ELA: F, Science: S
- Math: S, ELA: F, Science: F
- Math: F, ELA: S, Science: S
- Math: F, ELA: S, Science: F
- Math: F, ELA: F, Science: S
- Math: F, ELA: F, Science: F

So, there can only be 8 pigeonholes!

There are 9 people

So, it's true!

- What do we know … about grades for 3 tests?
- It's a pigeonhole problem …
  - So what are the pigeonholes?
  - What are the pigeons?

Can Wizard help Dorothy ?

-1	1	1	1	1	-1
1	-1	1	1	-1	1
1	1	-1	-1	1	1
1	1	-1	-1	1	1
1	-1	1	1	-1	1
-1	-1	1	1	1	-1

Wizard told Dorothy that he would help her to get home, if she could create a magic 6X6 square with entries either "+1" or "-1", so that all vertical, horizontal and diagonal sums would be different.

Can Wizard help Dorothy ?

Create a magic 6X6 square with entries either "+1" or "-1", so that all vertical, horizontal and diagonal sums would be different.

The pigeonholes are all possible sums of 6 numbers either ''+1" or ''-1".

The pigeonholes are then as follows
- 6: All numbers are 1's
- 4: 5 +1's, 1 -1's
- 2: 4 +1's, 2 -1's
- 0: 3 +1's, 3-1's
- -2: 2 +1's, 4 -1's
- -4: 1 +1's, 5 -1's
- -6: all numbers are -1's

So, there can only be 7 pigeonholes!

The Pigeon are 6 rows, 6 columns and 2 diagonals.

So there are 14 pigeons!

There must at least two sums are equal.

It is impossible to create such a magic square!!!

- What do we know … about sums?
- It's a pigeonhole problem …
  - So what are the pigeonholes?
  - What are the pigeons?

Generalized PP

Recall: If you place $n + 1$ objects in n holes, there is at least one hole must contain more than one (≥ 2) object.

Generalization: If you place n objects in k holes, there is at least one hole must contain

$$\left\lceil \frac{n}{k} \right\rceil$$

objects.   (ceiling: round up)

i.e., it is the smallest integer greater than or equal to n/k.

Examples: ceiling of 4/2 is 2, ceiling of 10/3 is 4; ceiling of 20/9 is 3.

There are 20 pigeons fly to 9 holes. Is there one hole must contain 3 pigeons? (ceiling of 20/9 is 3.)

Let us **suppose** that each hole contains less than 3 objects, i.e., each hole contains less than or equal to 2 objects; then the total number of objects would be less than or equal to 18, contradiction!

Back to Birthday

There are _____ students in the Math Circle.

At least _____ students have birthdays of the same month!!

What are the pigeonholes?

What are the pigeons?

What do we know?

PP or GPP?

There are _____ students in the Math Circle. At least _____ students have birthdays of the same month.

The pigeonholes are months.

The pigeonholes are then as follows
January	February
March	April
May	June
July	August
September	October
November	December

Goal: There is one hole have at least __ students.

Generalized Pigeonhole Principle!

- What do we know ... about months of birthday?
- It's a pigeonhole problem ...
  - So what are the pigeonholes?
  - What are the pigeons?

First Name Initials

There are _____ students in the Math Circle.

At least _____ students have the same first name initials!!

What are the pigeonholes?

What are the pigeons?

What do we know?

PP or GPP?

One octopus wants to visit UCF with wearing socks. He has a drawer with socks: each sock is a color featured on the UCF Knights Logo.

Question: Without looking, how many socks must he pick in order to have 8 socks of the same color?

The pigeonholes are colours: Golden and Black

Goal: have 8 pigeons in the same pigeonhole!

Can we do it?

- What do we know … about colors?
- It's a pigeonhole problem …
  - So what are the pigeonholes?
  - What are the pigeons?

Generalized PP Backwards

Generalized PP: If you place *n* objects in *k* holes, there is at least one hole must contain $\lceil \frac{n}{k} \rceil$ objects.

(ceiling: round up)

Generalized PP backwards: If at least one hole must contain an optimum number of objects, then you must have at least

objects = # boxes X (optimum − 1) + 1

Example: There are 9 boxes, and if at least one hole must contains 3 pigeons. The worst case, every hole contain 2 pigeons, so we need one more pigeons to make one hole has 3 pigeons!

So the total number of pigeons we need is

9X2+1 = 9 X (3-1) +1

Socks

Rachel has a drawer with 10 black and 10 white socks.

Without looking, how many socks need Rachel pull out to be sure that she has a pair of the same color?

What are the pigeonholes?

What are the pigeons?

What do we know?

PP, GPP or GPPB?

Rachel has a drawer with 10 black and 10 white socks. how many socks need Rachel pull out to be sure that she has a pair of the same color?

The pigeonholes are all possible colors: black and white.

The pigeons are socks!

Goal: require at least one hole having 2 socks!

Generalized PP backwards!

The minimum number of socks need to be pulled out is:
2X(2-1)+1= 3.

So Rachel needs to pull out 3 socks!

- What do we know … about grades for 3 tests?
- It's a pigeonhole problem …
  - So what are the pigeonholes?
  - What are the pigeons?

Colorful Socks

Rachel has a drawer with 10 red, 10 white, and 10 blue socks.

Without looking, how many socks need Rachel pull out to be sure that:

two socks of the same color?

two white socks?

two different socks?

Pet-Socks

There is a drawer with cat-socks (black or white). Presume that the cat has 4 legs, each of which needs a sock.

how many socks must be taken from the drawer to ensure that the cat has 4 socks of the same color?

What are the pigeonholes?

What are the pigeons?

What do we know?

PP, GPP or GPPB?

More Pet-Socks

What if there are cat-socks in all the colors of the rainbows, plus black and white.

how many socks must be picked now to guarantee 4 socks of the same color?

What are the pigeonholes?

What are the pigeons?

What do we know?

PP, GPP or GPPB?

8 colors: | R | O | Y | G | B | P | B | W |

Back to Octopus

One octopus has a drawer with socks: each sock is a color featured on the UCF Knights Logo.

Without looking, how many socks must he pick in order to have 8 socks of the same color?

What are the pigeonholes?

What are the pigeons?

What do we know?

PP, GPP or GPPB?

One octopus has a drawer with socks: each sock is a color featured on the UCF Knights Logo. Without looking, how many socks must he pick in order to have 8 socks of the same color?

The pigeonholes are colours: Golden and Black

The pigeons are socks!

Goal: have 8 pigeons in the same pigeonhole!

Generalized PP backwards!

The octopus needs to pick **2X(7-1)+1=15** socks!

Have Fun with Socks

Squid studied Octopus's mittens.
"Your mittens are nice," he finally admitted.
"Those are great socks," said Octopus.
"Should we . . . share?" asked Squid.

A splendid exchange of socks and mittens followed.