Compute the probability that a randomly chosen positive divisor of 10^99 is an integer multiple of 10^88.

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Probability = the number of desired outcomes / by the number of total outcomes.

First, to calculate the probability of the problem, we need to find the number of desired outcomes and total outcomes.

Finding the number of total outcomes:

*Since a property of exponents states when there are two numbers a and b, (ab)^n = a^n * b^n, therefore, 10^99 can be shown as 2^99 · 5^99. From this we can see that any divisor of 10^99 must be a product of some number of 2's, between 0-99, and some number of 5's, between 0-99. Considering the fact that the product of 0-99 number of 2's and 0-99 number of 5's are all possible combinations, to find the exact number of combinations, we can write the equation: x = (99+1)(99+1), where x is the number of combinations. We get this equation using the counting principle which states that there are p possibilities for one event and q possibilities for another event, then the number of possibilities for the events is p · q. For that reason, we get the equation x = (99+1)(99+1). In this case, we added a 1 to each 99 to include the number 0 as a possibility, then, the number of possible 2's and 5's are multiplied to get 10,000. So there are a total of 10,000 outcomes.*

**Basically, to calculate how many divisors a number has, find the prime factorization, add 1 to all the exponents, and then multiply them together.*

Finding the number of desired outcomes:

To find the number of desired outcomes, we need to find the number of positive divisors of 10^99 that are also multiples of 10^88. Since the bases are both 10, we can just focus on the exponent numbers 99 and 88. If we want to find numbers that are both multiples of 10^88 and divisors of 10^99, the only possible solutions are for the exponents to be 88-99. This is because, if we go under 10^88, it wouldn't be a multiple of 10^88 anymore and if we go above 10^99, it wouldn't be a divisor of 10^99 anymore. Thus, using the same way to find the number of total outcomes, we can find the number of desired outcomes by creating the equation x = (11+1)(11+1)= 144. So the number of desired outcomes is 144.

Therefore, the probability that a randomly chosen positive divisor of 10^99 is an integer multiple of 10^88 is 144 / 10,000 = 9 / 625