# Patterns in mathematics and nature

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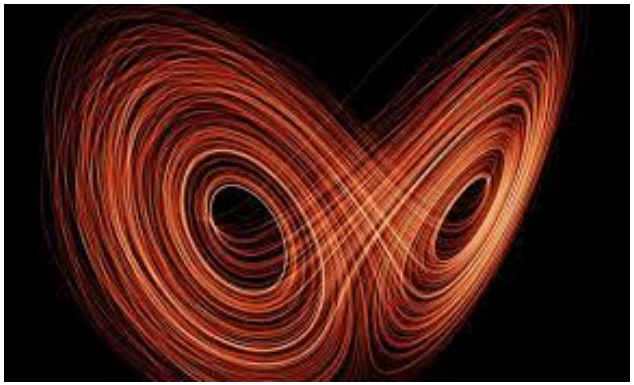

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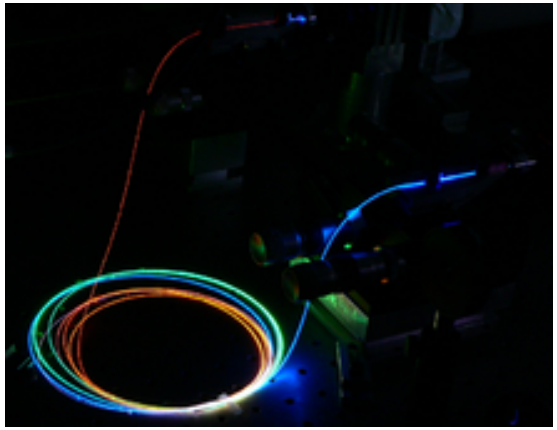

# A little bit about myself...

- Currently I am a Postdoctoral Scholar in the Department of Mathematics at the University of Central Florida (UCF) under the supervision of Professor Alexander Tovbis.
- In the fall I had the pleasure of visiting the Mathematical Sciences Research Institute (MSRI) in Berkeley, CA as a Postdoctoral Fellow under the supervision of Professor Percy Deift.
  - Semester program on random matrix theory and interacting particle systems.
  - `https://www.msri.org/web/cms`
- I received a Ph.D. in mathematics from The State University of New York at Buffalo (2021) under the supervision of Professor Gino Biondini.
- My research focuses on the rigorous mathematical analysis of integrable nonlinear wave equations.

# Outline

- Number theory
  - Pythagorean triples
  - Primitive Pythagorean triples
- Sequences
  - Fibonacci sequence
  - Fibonacci spiral
- Probability
  - Random walks
- Complex numbers
  - Mandelbrot set
  - Self-similarity

# Pythagorean triples

- A Pythagorean triple is a set of three positive integers $a$, $b$, and $c$, such that
$$a^2 + b^2 = c^2\,, \tag{1}$$
and is commonly denoted by $(a, b, c)$.
- The name comes from the Pythagorean theorem which states that the sides of a right triangle satisfy $a^2 + b^2 = c^2$. Thus, Pythagorean triples are the right triangles with integer side lengths.
- A primitive Pythagorean triple is one in which $a$, $b$, and $c$ are coprime (they have no common divisor larger than one)
- Pythagorean triples can be calculated using Euclid's formula. The formula states that for any pair of integers $m$, $n$ with $m > n > 0$, the integers
$$a = m^2 - n^2, \quad b = 2mn, \quad c = m^2 + n^2 \tag{2}$$
form a Pythagorean triple.
- Pythagoras and Euclid were ancient Greek philosophers.

# Pythagorean triples

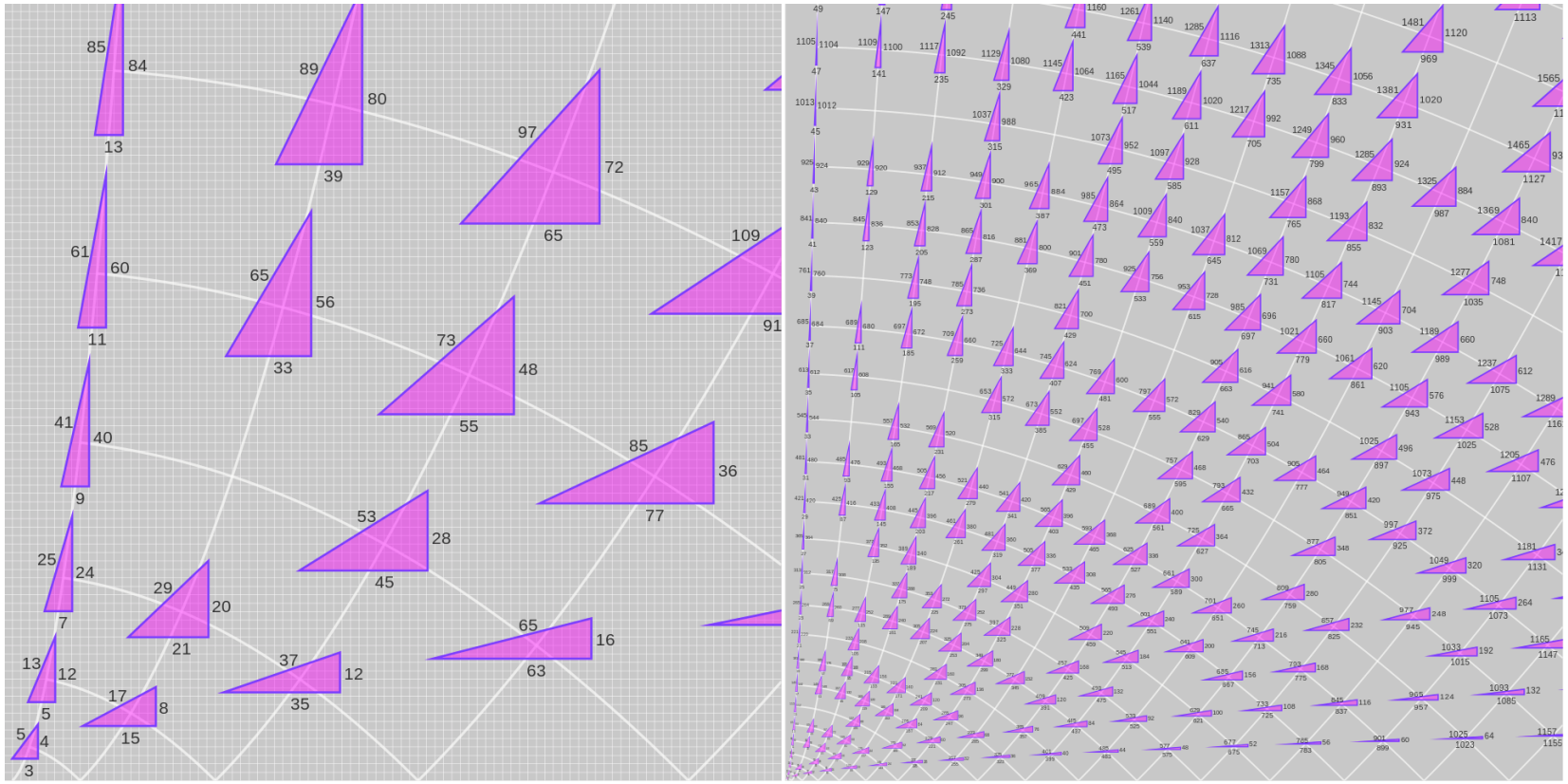

Figure: By Adam Cunningham and John Ringland

- The odd leg $a$ is plotted on the horizontal axis, and the even leg $b$ is plotted on the vertical.
- The curvilinear grid is composed of curves of constant $m - n$ and $m + n$ in Euclid's formula.

# Prize problems

- Question: Find gcd(238,1020) (greatest common divisor).
- Question: Recall Euclid's formula for a Pythagorean triple, i.e.

$$a = m^2 - n^2, \quad b = 2mn, \quad c = m^2 + n^2,$$

for integers $m$,$n$ with $m > n > 0$. When both $m$ and $n$ are odd $(a, b, c)$ is not a primitive Pythagorean triple. Why?

# Fibonacci sequence

In mathematics, a sequence is an ordered list of numbers (possibly infinite) where repetition may occur. An integer sequence is a sequence of integers.

Fibonacci sequence:

$$F_0 = 0\,, \quad F_1 = 1\,, \text{ and } \; F_n = F_{n-1} + F_{n-2} \quad \text{for } n > 1\,. \tag{3}$$

- The Fibonacci sequence appears first in Indian mathematics.
- The sequence is named after the Italian mathematician Fibonacci (aka Leonardo of Pisa) who used it to study the growth of rabbit populations.
- Golden ratio: $F_n = \frac{\phi^n - \psi^n}{\phi - \psi}$, where $\phi = \frac{1+\sqrt{5}}{2}$ is the golden ratio.
- Amazingly this sequence is found throughout nature!

# Fibonacci sequence in nature

Figure: Spectra Magazine (Left) flower and (Right) shell

Figure: (Left) Galaxy and (Right) Hurricane

# Prize problems

- Question: Recall the Fibonacci sequence

$$F_0 = 0, \quad F_1 = 1, \quad F_n = F_{n-1} + F_{n-2}, \quad n > 1.$$

What is $F_{12}$?

# Random walk

- In probability theory a random process is a mathematical object which serves as a model for phenomena that appear to vary randomly over time.
- A random walk is a type of random process that consists of a succession of random steps.
  - The term "random walk" was introduced by Karl Pearson in 1905.
- Applications:
  - Brownian motion (path traced by a molecule as it travels in a liquid or gas)
  - Stock price fluctuations
  - Neuroscience (model cascades of neuron firings in the brain)
  - Computer science (random walks used to estimate the size of the Web)

# Random walk

Figure: Antony Gormley's "Quantum Cloud" sculpture in London designed by a computer using a random walk algorithm

# Madelbrot set

Consider the complex function:

$$f_c : \mathbb{C} \to \mathbb{C}, \quad f_c(z) = z^2 + c\,. \tag{4}$$

Madelbrot Set =

$$\{c \in \mathbb{C} : \text{sequence}\, f_c(0),\, f_c(f_c(0)),\, \text{etc. remains bounded}\}\,. \tag{5}$$

Note:

$$f_c(0) = 0^2 + c = c,$$
$$f_c(f_c(0)) = f_c(c) = c^2 + c, \text{ etc.}$$

- Discovered by Robert W. Brooks and Peter Matelski.
- Benoit Mandelbrot obtained high-quality visualization while working at IBM's Watson Research Center.

# Mandelbrot set

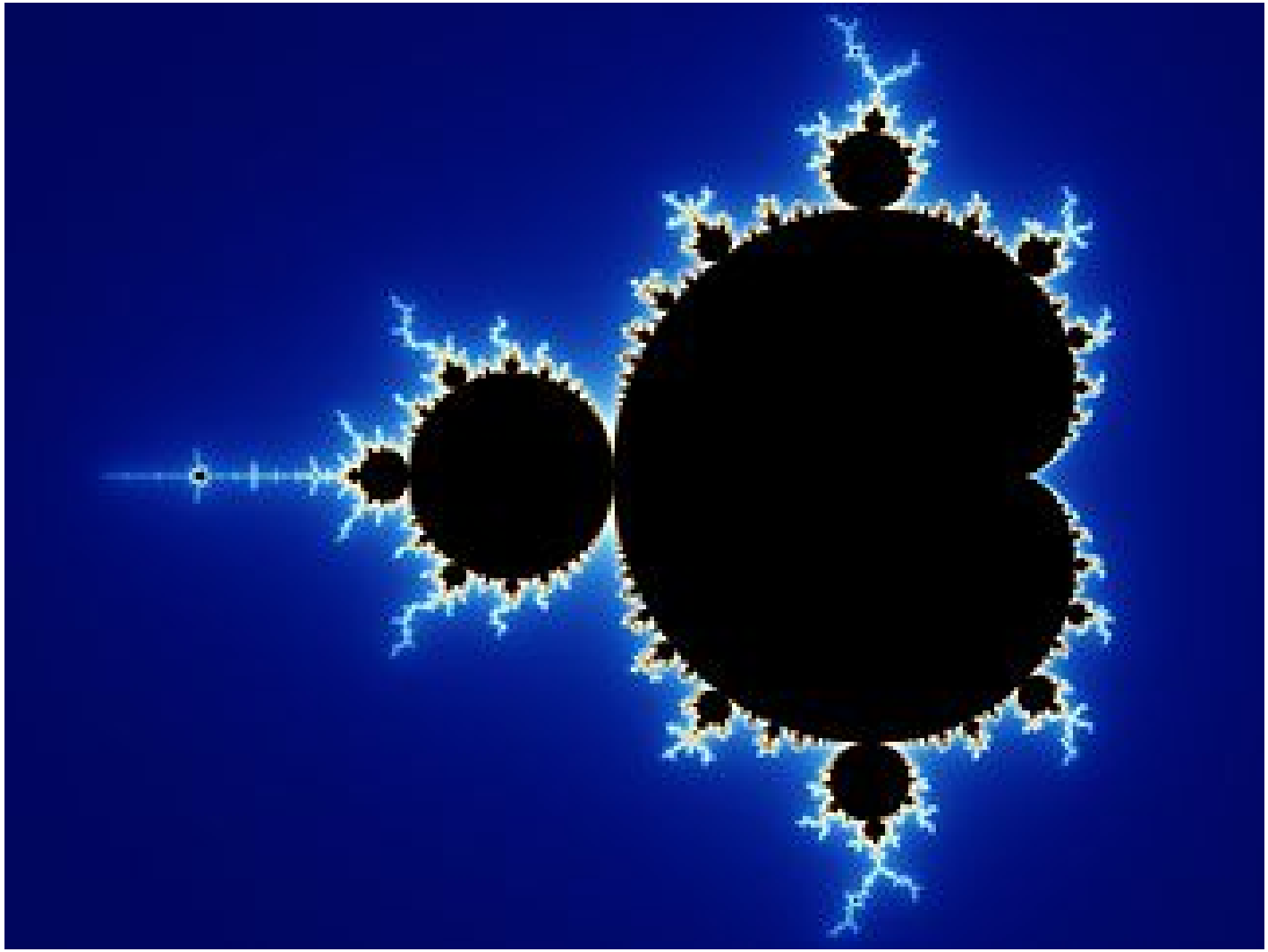

Figure: By Wolfgang Beyer with program Ultra Fractal 3

# Prize problems

- Question: Evaluate $f_c(f_c(f_c(0)))$ where $f_c(z) = z^2 + c$.
- Question: Is the sequence $\frac{n}{n^2+1}$ where $n \geq 1$ bounded? Justify your reasoning.