Advanced Group Math Circle

Assorted Challenge Problems

Benjamin Keene

UCF

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Introduction

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Practicing your mathematical skills in various different topics helps hone your intuition, and strengthens your ability to tackle new problems you have no experience with.

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v. Afterwards, we'll discuss how to answer that question.

Hint 1

I'll give you a hint before each question. Here is the first hint.

$$a > b \implies \frac{1}{a} \underbrace{?}_{< \text{ or } >} \frac{1}{b}$$

Question 1

Which is greater: $\dfrac{2020}{2021}$ or $\dfrac{2021}{2022}$?

Hint 2

How can you compare the following two numbers:

$$(3 \times 4 \times 100 \times 100) \quad \text{versus} \quad (2 \times 6 \times 100 \times 100)$$

Question 2

Which is greater: 333333×444444 or 222222×666667,
and by how much?

Hint 3

Calculate the following perfect squares, do you notice anything interesting about them?

$$23 \times 23 =$$
$$11 \times 11 =$$
$$111 \times 111 =$$
$$222 \times 222 =$$

Question 3

Does there exist a perfect square that ends in 9 distinct digits?

Hint 4

I have the following five integers:

$$\{-5, -2, 8, 12, 20\}.$$

Pick three at random, what do you notice about the sign of the sum of those three integers?

Question 4

Suppose you have a set of 2002 integers, and the sum of any 100 of them is positive.

Explain why the sum of all 2002 integers *must* be positive.

Hint 5

How much smaller is $1 + 3 + 5 + 7$ than $2 + 4 + 6 + 8$, without adding them up and comparing?

Question 5

Without using the sum of integers formula, explain which is greater:

- The sum of all even integers from 0 to 100,
- The sum of all odd numbers from 1 to 99.

By how much is it greater than the other?

Hint 6

You and your friends have started collecting Pokémon cards. Each of you owns a different number of Pokémon cards.

If you add up the number of cards everyone owns, what is the minimum possible total?

Question 6

A group of 15 friends have collected a total of 100 Pokémon cards.

Show that 2 students own the same number of Pokémon cards.

Hint 7

You have nine pairs of socks in your dresser (and you have two feet). If the sun has gone out and we live in darkness, how many socks must you blindly grab from your dresser to guarantee you have at least one matching pair of socks?

Question 7

Suppose you have a collection of 11 natural numbers.

Show that there must be two numbers whose difference is divisble by 10.

Hint 8

How can the following expression be factored?

$$a \cdot b \cdot c + (2a) \cdot (2b) \cdot (2c)$$

Question 8

Simplify the fraction

$$\frac{1 \cdot 2 \cdot 3 + 2 \cdot 4 \cdot 6 + 4 \cdot 8 \cdot 12 + 7 \cdot 14 \cdot 21}{1 \cdot 3 \cdot 5 + 2 \cdot 6 \cdot 10 + 4 \cdot 12 \cdot 20 + 7 \cdot 21 \cdot 35}.$$

Hint 9

If I give you any natural number n, what is the next consecutive natural number?

Question 9

Can the sum of four consecutive natural numbers be divisible by 4?