The Extremal Function for $K_{10}$ Minors

Robin Thomas, Dantong Zhu*, Georgia Institute of Technology

For positive integers $t$ and $n$, the maximum number of edges that an $n$-vertex graph with no $K_t$ minor can have is known as the extremal function for $K_t$ minors. In 1968, Mader proved that for every integer $t = 1, 2, \ldots, 7$, a graph on $n \geq t$ vertices and at least $(t - 2)n - \binom{t - 1}{2} + 1$ edges has a $K_t$ minor. Jørgensen showed that a graph on $n \geq 8$ vertices and at least $6n - 20$ edges either has a $K_8$ minor or is isomorphic to a graph obtained from disjoint copies of $K_2,2,2,2,2$ by identifying cliques of size 5. Song and Thomas further generalized the results for $K_9$ minors. The extremal functions for $K_t$ minors where $t \leq 9$ have been important for proving several results related to Hadwiger’s conjecture. In this talk, I will discuss our work-in-progress on the extremal function for $K_{10}$ minors.