Peer instruction methodology for linear algebra subject: a case study in an engineering course

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***Abstract*— While the initial course in linear algebra plays a decisive role in an engineer's basic formation, it is often marked by high rates of student failure and dropout and thus constitutes an important issue in the field of engineering education. The present study aims to enhance the teaching and learning processes in the first phase of linear algebra in engineering curriculum, viz., matrices, systems of linear equations, notions of vectors, and line and plane equations. To address this end, a methodological teaching proposal, Peer Instruction (PI), is proposed to improve student comprehension, classroom interaction, cooperation among peers, and assimilation of fundamental concepts. The PI method was applied in the initial phase of a linear algebra course in the undergraduate Chemical Engineering program at the Federal University of Ceará, Brazil. To measure student prior knowledge, a conceptual pre-test was conducted. Subsequently, eleven 2-hour classes were taught using PI, and a final exam was then conducted. Comparison of pre- and post-test results shows significant increase in student performance. The incremental increase in student-teacher and student-student interactions played a significant role in methodology's success.**

Keywords—Peer instruction; linear algebra; engineering education; active learning; previous knowledge.

I. INTRODUCTION

Failure rates among first-year engineering students are increasingly encountered in universities worldwide [1]. A contributing factor to these high rates is the inadequacy of the students' academic background in mathematics [2].

In general, students have difficulty in relating previous knowledge to new concepts that will be learned at university. In this regard, basic abstract concepts in linear algebra, a core course in the engineering curriculum, merit special attention. Linear algebra provides a foundational language for mathematical modeling of a broad number of problems. Thus mastering these concepts is crucial to the education of future engineers [3].

Such basic concepts from secondary education such as matrices, systems of linear equations, notions of vectors, and line and plane equations, form a set of prerequisites for the proper understanding of linear algebra [4]. Accordingly, it is critical that students master these concepts in order to develop their abstract thinking in linear algebra; otherwise major difficulties are certain to arise [5-6]. Providing an appropriate background to students should help them overcome barriers between the abstract and real worlds. Such initiatives foster a better understanding of linear algebraic concepts to the benefit of students' academic studies and subsequent professional careers.

The conventional teaching method, based on lectures given by an instructor addressing a class of passive students, has proven ineffective for linear algebra courses. It fails to motivate students and makes it more difficult for them to absorb abstract concepts, as evidenced by the unsatisfactory learning rates found in evaluations [5].

The present study applies a teaching method, Peer Instruction (PI) [7], to support alternative teaching and learning processes for fundamental concepts of linear algebra in the context of the engineering curriculum. PI's immediate benefits include increased motivation and interest in the subject.

The PI method was developed by Eric Mazur, a professor of physics at Harvard University [7]. Its objectives are to motivate students' class participation, their collaboration with their peers, and, in particular, their assimilation of fundamental concepts in linear algebra.

The PI methodology was implemented throughout the Linear Algebra course taught in the program of Chemical Engineering at Federal University of Ceará (UFC). The purpose of this study, however, is to analyze student performance improvement solely in the initial phase of the course, in which basic mathematical concepts, viz., matrices, systems of linear equations, notions of vectors, and line and plane equations, are taught. Since these concepts customarily

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are part of the secondary curriculum, students are expected to have previous knowledge of them. This is why analyzing the performance data restricted to the initial phase of the course is an effective way to assess PI's success.

Accordingly, it should be understood that references to the initial phase of the linear algebra course encompasses the previous knowledge acquired in secondary education.

The study finds that using different materials and alternative instruction strategies, based on PI, in teaching fundamental concepts of linear algebra enhances students' learning experience. This is particularly meaningful in regard to the initial phase of the course. Mastering these foundational concepts has a decisive impact in assimilating abstract ideas from the latter part of the course. Increased motivation has also been observed.

The study is divided into four principal parts. The introductory section consists of a brief history Peer Instruction and its systematization, while the second describes the implementation of PI techniques in teaching the initial phase of a linear algebra course. The third section presents the study's results and analysis, and the concluding section offers some thoughts about the study's result, implications, and further dimensions to be explored.

II. Peer Instruction

In the early 1990s, Mazur developed Peer Instruction as a teaching method to help his students improve learning in his introductory physics course. In view of its effectiveness [8-10], PI is now used in diverse courses in many countries [11-16]. Despite its international success, however, PI has not been sufficiently explored in linear algebra instruction in Brazil. In this study, Didactic Engineering (DE) [17] is combined PI to develop an efficient mechanism to teach the first quarter of linear algebra courses. This is the principal innovation presented in these pages.

PI's fundamental objective is to promote student engagement and cooperation in class, as well as the assimilation of fundamental concepts. The conceptual idea behind the method is to induce students to appreciate the importance of discovering for themselves the bases of theories covered in the classroom, thus honing their abstract thinking skills.

A PI lecture organizes its contents under critical topics. For each key concept, a brief explanation of about 10 minutes is provided, followed by a conceptual question using a multiple-choice format. Students are encouraged to think through their answers for 1 to 2 minutes, and then respond by a show of hands, flashcards, or electronic devices (clickers) directly connected to the instructor's computer. Subsequently, the instructor registers their responses for comparative purposes.

If at least 70% of the students select the correct answer, the instructor briefly discusses it without taking a further poll. As a rate below 30% implies students misunderstood the concept, the instructor explains it in further detail, often using another approach. Subsequently, the instructor poses another conceptual question. Ideally, students should answer it correctly between 30 and 70 percent, implying that the question is challenging but not out of reach. In this situation, students are asked to discuss their responses with their peers, and endeavor to convince them to adopt their own answer. Simultaneously, the instructor should move around the classroom to foster fruitful discussions.

After two to four minutes, it is time for the next round of polling; if most students select the correct answer, the instructor can move on to the next topic; otherwise, he or she may elect to explain the question, analyzing each response. Fig. 1 summarizes the PI process [10].

Another important aspect of PI is that it requires students to prepare for class beforehand. Students must read material assigned by the instructor before coming to class. Such material might include text, videos, or bibliographic references that re-enforce the basic concepts to be discussed in the next class. Just-in-Time Teaching, developed by Novak, Gavrin, Christian, and Patterson [18], is used to incentivize such reading. Students are required to study the material, answer questions related to the complexities of the concepts, and provide their responses to the instructor by the deadline. For example, the instructor might ask, "Did you find anything difficult or confusing in the reading? If not, what did you find most interesting?" The goals of reading assignments are to encourage students to prepare for class, bolster student motivation, and provide the instructor sufficient time to adjust in-class activity to address students' difficulties.

As Mazur's experience confirms, Peer Instruction's successful implementation does not depend on the polling system. Fundamentally, PI is a student-centered teaching methodology that emphasizes the concepts of and provides a proper environment for peer discussion and collaboration. The instructor plays a role as a facilitator in the teaching-learning process, helping students' through their difficulties [19].

Vygotsky's theory empathizes that cognitive development is based on social interaction [20]. This concept is embodied in the Peer Instruction method, since the instructor, who plays the role of mediator in the learning process, seeks to stimulate student interaction. This enables student to actively participate and effectively cooperate with their counterparts, thereby increasing their own comprehension. These initiatives yield teacher-student interaction, student-student interaction, and a new interaction with previous knowledge.

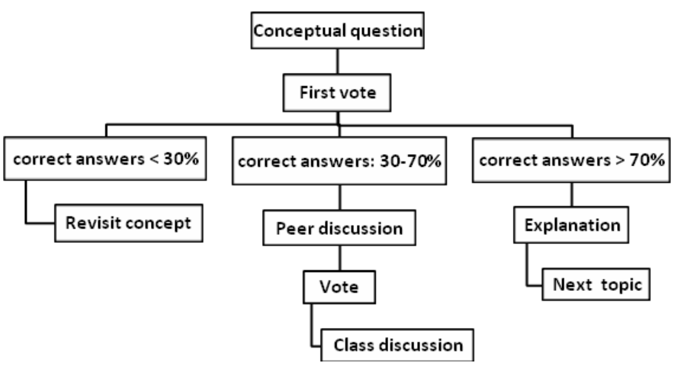

Fig. 1. Peer Instruction implementation process.

According to Ausubel's learning theory, to attain new knowledge, students must link it to relevant facts they already know [21]. When a new concept relates to existing knowledge gained in the classroom, the student will be better equipped to transform the knowledge into meaningful learning to be applied outside that milieu. Reading assignments have provided students the prior knowledge required for the absorption of a new concept. Hence, this methodology favors learning where students connect new content to pre-existing knowledge thereby generating meaningful learning.

III. METHODOLOGY

This study is based on the Peer Instruction method. The Just-in-Time Teaching method was applied as a reading incentive and the Didactic Engineering, methodology developed by Michèle Artigue [17], provided support in the organization, observation, and analysis required to prepare classes [22].

A. *Learning Outcome and Course Description*

The purpose of the lectures is to provide students with a sound background to the concepts of the initial phase of the linear algebra course. Their goal is to enable students to identify and solve linear equations, using matrix systems, highlighting geometric associations and physical interpretations. The reading assignments and conceptual and test questions are designed to meet expected learning outcomes described in Table I.

The study was conducted in a linear algebra course in the Chemical Engineering program at the Federal University of Ceará. Of the 70 students enrolled in the course, 64 were taking it for the first time. Student ages ranged from 17 to 21 years. There were 22 females and 58 males in the class.

The two-hour class met twice weekly. The lectures comprised 64 hours, of which 20 were dedicated to covering fundamental concepts required for the study of linear algebra. Table II shows the topics covered during the lectures.

TABLE I. EXPECTED LEARNING OUTCOMES

	Following the respective lecture, students should be able to:
S1	Identify types of matrices
S2	Operate matrices
S3	Distinguish between matrix and real number properties
S4	Solve linear equations using matrix systems
S5	Compute determinants of matrices and understand their importance in solving linear systems and inverting matrices
S6	Geometrically interpret solutions of linear systems via vectors, lines, and planes
S7	Write elementary mathematical proofs.

TABLE II. COURSE DESCRIPTION (INITIAL PHASE)

Lecture	Description
1	Explanation of study's methodology; Diagnostic pre-test
2	Matrix operations
3	Systems of linear equations Homogeneous systems
4	Row reduction and echelon forms Gaussian elimination
5	Inverse matrices and their applications
6	Determinants
7	Adjugate matrices
8	Vectors in n-space
9	Line and plan equations
10	Post-test

B. *Peer Instruction Implementation*

On the first day of class, the study's methodology and how it would be implemented in the class were introduced to students. The importance of their active participation and the need for pre-class reading was highlighted. Students reacted positively to the study's methodology and implementation.

A diagnostic pre-test, consisting of 10 true-or-false questions, was conducted to evaluate students' previous knowledge of basic concepts critical to the study of linear algebra. The same test was re-conducted subsequent to the lectures to assess student assimilation of these concepts.

Following the Just-in Time Teaching protocol, the reading material and assignment related to the content of their next class was periodically sent students via the university's Virtual Learning Environment, Sistema Integrado de Gestão de Atividades Acadêmicas (Integrated Academic Activities Management). The material provided was carefully designed to effectively and efficiently summarize a broad spectrum of online literature and content [23-27]. As an incentive to completing the pre-class reading, the assignments constituted 10% of the student's final grade. This credit was received by student solely on the basis of effort. An exam covering a significant number of conceptual questions was administrated at the conclusion of the initial phase.

The JiTT method recommends that students submit their completed reading assignment prior to class. However, such a task would hardly be in practical due to local constrains. Alternatively, the instructor used an alternative protocol from the Didactic Engineering methodology [17].

The students' completed assignments were received at the beginning of, rather than prior to, class. To compensate for this discrepancy, in preparing the lecture, the instructor predicted probable difficulties in assignment completion, based on teaching experience and current literature. This strategy, viz., a priori analysis, is part of DE's second phase [17].

In accordance with the final stage of DE, viz., a posteriori analysis and validation, following each class, completed assignments were analyzed by the instructor and compared with the anticipated results. Drawing on this analysis, the instructor began the next class with a review of content inadequately assimilated as evidenced by the completed assignments.

Guided by the PI method, at the beginning of class, each student received a set of four cards for use in the polling process. After a brief discussion of the reading assignments and supporting material, the lecture main topic was divided into subtopics, followed by conceptual questions selected to advance the students' critical reasoning. The questions and multiple-choice answers were adapted from books and websites related to the course. Each question covers a specific concept or a property to be discussed in class [14, 23-26].

Following the PI strategy, the instructor presents a brief explanation of the content and poses a conceptual question to initiate the polling process. Students respond to the question without consulting textbooks or any other external sources.

If 30% to 70% of students answer a given question correctly, peer discussion in small groups of approximately two to three students is initiated. The groups are assigned with the goal of confronting students with different answers. After about 4 minutes of group discussion, the instructor polls the class again, explains the correct answer, and presents the next component of the lecture.

On the other hand, if the initial poll indicates that less than 30% of the class answered the question correctly, the instructor briefly explains the concept further, using a different approach. Subsequently, the instructor discusses each response to the question, guiding them to analyze them appropriately and thus arrive at the correct answer.

In the case where the rate of correct answers exceeds 70%, the instructor may elect to re-poll the class, following a peer group discussion as previously described, rather than simply moving on to the next topic. When this route is taken, the percentage of correct answers in the second poll generally surpasses 90%. This clearly indicates that the class has generally assimilated the lesson and there is no need for further clarification.

Student evaluation was performed continuously throughout the course. Student grades were based on reading assignments, class participation and the post-test. To validate the evaluation of the PI methodology, the test content was identical to that of the pre-test administered at the course's onset. Comparison of the results of the pre- and post-tests yields an accurate analysis on the progress of a student's knowledge and the success of the instructional method.

IV. Results

In this section, the results obtained from the implementation of Peer Instruction during the initial phase of a university course in linear algebra are presented and analyzed.

A. Conceptual Questions

During the initial phase of the linear algebra course, a total of 20 conceptual questions were posed. To build student confidence and motivate them to participate in class, the difficulty level of the questions was gradually raised. Student responses were compiled on a worksheet for future analysis.

Of the 20 conceptual questions posed in this period, only 3 fell below the 30% threshold for correct responses in their initial poll. On the other end of the spectrum, 6 questions exceeded the 70% bound. Thus 55% of the questions fell within the 30% to 70% of range of correct responses sought by the PI method in their first round.

Review of responses indicated that questions involving matrix multiplication properties generated considerable doubts and discussion among students. Table III exemplifies one of those questions. Its initial polling generated a correct response rate of just 10%. An analysis of each potential response was carried out. Most students did not distinguish between the rules governing matrix multiplication and those applicable to the arithmetic multiplication of real numbers. This is a basic anticipated ability, as noted in Table I, S3. The question prompted a particularly stimulating discussion among the class, intensifying student's buy-in and motivation for active, collaborative participation in the course.

Student collective engagement in solving problems was not limited to this question. Indeed their increasing integration in addressing all questions was evident throughout the e period studied. Within a stimulating atmosphere, it was common to witness students attempting to convince their peers that their responses were correct. The increase in correct responses after discussion is illustrated in Fig. 2.

Peer discussions reflected progressive improvement in the rate of correct answers, as students became more confident in explaining their ideas to one another. A cooperative atmosphere pervaded the classroom.

B. Reading Assignments

In the initial phase of the course seven reading assignments covering matrices, linear equations systems, vectors, lines, and planes equations were completed.

Each assignment began with the question: "What did you find difficult or confusing about the reading? If nothing was difficult or confusing, what did you find most interesting?" This helped the instructor identify the principal challenges facing students.

Despite expected previous knowledge concerning these topics, students encountered substantive difficulties as evident in their completed pre-class assignments. A potential contributing factor could well be deficiencies in abstract mathematical reasoning.

TABLE III. CONCEPTUAL QUESTION ON MATRIX PROPERTIES. (correct answer :d)

If a square matrix A is such that $A^2 - 4A^3 = 0$, then,
a) $A^2(1-4A) = 0$
b) $A = 0$ or $A - 4A^2 = 0$
c) $\frac{A^2}{A^3} = 4$
d) none of the above.

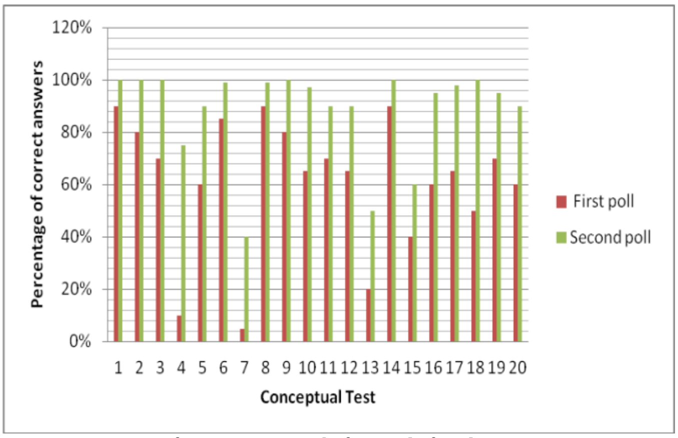

Fig. 2. Percentage of correct answers before and after discussion.

In this regard, particular attention should be accorded a question testing math proof testing ability presented in Table IV (see also, Table I, S7). An overwhelming majority of 90% of students failed to find the correct solution. In light of this result, the instructor provided another explanation of the concept, using a different approach, but not providing the solution. The question was then resubmitted as a reading assignment. In this second round, 75% of the students answered the question correctly. To re-enforce the learning experience, the instructor asked a student to present their solution on the blackboard. Students reacted quite positively.

C. *Assessment of Student Learning*

The diagnostic pre-test was administered to 60 students (85% of all enrolled students) on the first day of class. The test, described in Table V, consisted of 10 statements covering the secondary-school level concepts integral to understanding linear algebra. Students were asked to rate each statement as follows: T (true), F (false), DC (unknown content), NE (never studied) or NC (have studied content but cannot solve) content). For the purposes of this analysis, the last three designations were regarded as blank responses.

The data, depicted in Fig. 3, indicate that nearly 65% of students missed or did not know the answers of the statements, indicated a low-level of previous knowledge of these critical concepts. None of the questions produced a correct response rate above 57%. For the most essential topic, i.e., vectors and lines, only 19% of the students provided correct answers.

The same questions used as the diagnostic pre-test were given as the post-test at the end of the study period. A total of 58 students took both tests, and their aggregate data were used to compare student performance in the pre- and post-tests.

TABLE IV. READING ASSIGNMENT WITH VERY HIGH FAILURE RATE

Given a square matrix A and λ a scalar. Show that ? if and only if $Ax = \lambda x$ for some vector

TABLE V. **PRE-TEST APPLIED IN A STATEMENT FORMAT**

Question	Concepts	Statement	Goal[a]
1	Transpose matrix	The transpose of a lower triangular matrix is a lower triangular matrix.	S1
2	Multiplication of matrices	The product of the matrices A(2 x 3) and B(3 x 2) is a square matrix 2 x 2.	S2
3	Properties of the matrix product	If a square matrix C is such that C^2-C=0, then C(C-1)=0.	S2;S3
4	Inverse matrix	If A is an invertible matrix, then A^{-1} = I/A where I is the identity matrix.	S3;S5
5	Determinant	det(AB)=det(BA), where A and B are square matrices.	S5
6	Geometric interpretation of a linear system	A linear system of two equations and two variables does not have solution when their graphs are parallel lines.	S4
7	Linear system	A linear system of three equations and five variables has solution.	S4
8	Vector	The length of every vector is a positive number.	S6
9	Line	The intersection between two planes is a line.	S6
10	Plane	The intersection of three planes cannot be a single point.	S6

a. Goal Learning according to Table I.

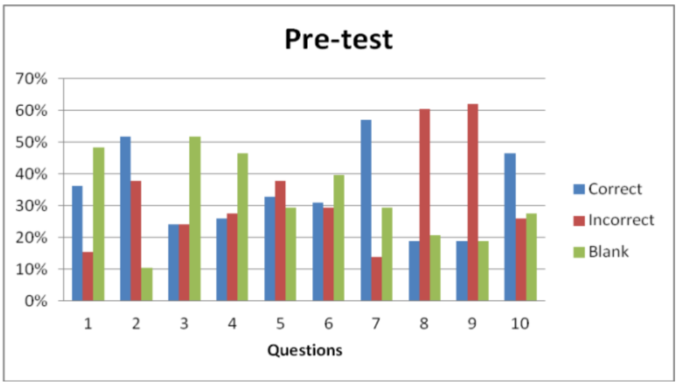

Fig. 3. Answers given to pre-test.

Fig. 4 portrays the rate of correct answers in the pre- and post-tests. It shows a significant post-test increase in correct responses for each question. Question 4, e.g., generated a 60% increase in its correct response rate. In regard to vectors and line equations, the most critical questions in the pre-test, post-test responses represented increases in correct responses rates of 38% and 33%, respectively.

The statistic t-test was used to determine whether there was a significant difference in students' pre-test scores, prior to the course's onset, and their post-test scores, following the initial phase of their instruction. As shown in Table VI, the p-value is 0.000 and t is 13.380. Since $p<0.05$, the results are statistically significant. In other words, there was a significant increase in average score in student performance.

TABLE VI. MEAN ANALYSIS OF PRE- AND POST- TEST

	N (sample)	Mean	Std Deviation	t-test[a]	Sig (2-tailed)
Pre-test	58	3.293	1.5448	-13.840	.000
Post-test	58	6.638	1.4228		

a. Result obtained using SPSS software.

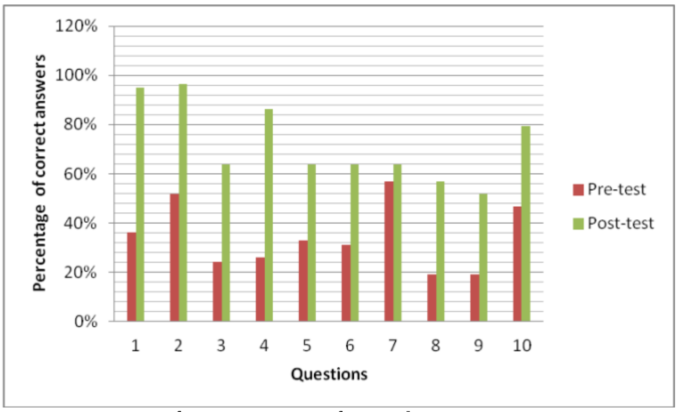

Fig. 4. Frequency of correct answers of pre and post tests.

Fig. 5 illustrates the overall result of students' performance in the pre-test and in the post-tests taking into account the frequency of the total number of correct, incorrect, and blank responses for each question.

The results of this comparison show a considerable improvement in student performance. As illustrated by Fig. 5, there was a 38% increase in the number of correct responses, a 6% decrease in incorrect responses, and a 31% decrease in the number of blank responses. The latter figure demonstrates the significant growth in knowledge acquired by students in the linear algebra course using the Peer Instruction methodology.

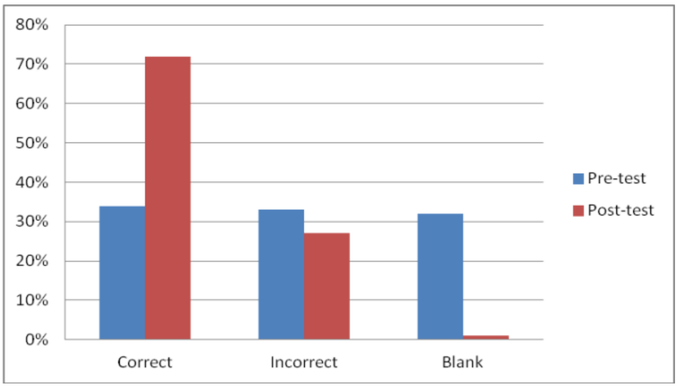

Fig. 5. Aggregate test results

V. CONCLUSION

Students were constantly encouraged to participate in class and share with others their solutions to the questions and the manner in which they arrived at them. Their positive response to such encouragement and their active participation enabled them to enrich their understanding of critical concepts in linear algebra and to discuss them with confidence with their peers, thus contributing to their educational progress.

The results further demonstrate that the Peer Instruction methodology contributes to students' development of their abstract thinking capacities and facilitates collaborative learning, skills which will better equip them for their professional careers. This stands in contrast to the record of traditional approaches to education that too often regard students as passive recipients of infused content.

Research covering the methodology should continue to be developed to systematize its classroom application. In this regard, in a forthcoming work, the authors will present further analysis of the results noted herein and conduct a study of more complex conceptual questions. In addition, an evaluation of PI's application throughout the entire course will be undertaken and the perspective of students to this innovational instructional methodology will be analyzed.

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