PRIMUS
Problems, Resources, and Issues in Mathematics Undergraduate Studies

ISSN: (Print) (Online) Journal homepage: https://www.tandfonline.com/loi/upri20

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To cite this article: Katiuscia Costa Barros Teixeira (2023) Pedagogical Strategies to Enhance Learning in a Linear Algebra Course, PRIMUS, 33:2, 152-174, DOI: 10.1080/10511970.2022.2048928

To link to this article: https://doi.org/10.1080/10511970.2022.2048928

PRIMUS
2023, VOL. 33, NO. 2, 152–174
https://doi.org/10.1080/10511970.2022.2048928

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ABSTRACT
In this study, we evaluate an innovative method to improve the teaching of linear algebra. The use of peer instruction in conjunction with a seminar strategy and supported by didactic engineering is proposed as a means to facilitate the mastery of abstract concepts, the undertaking of innovative research in problem solving, and the practical application of educational concepts. To assess the method's effectiveness, the Students' Evaluation of Educational Quality (SEEQ) questionnaire was administered and assessed. In addition, to determine whether this new teaching approach leads to better learning results than the traditional method, a comparison between control and experimental groups was conducted. The results indicate that the mixed method creates a more dynamic learning environment for students than the traditional method, contributes to students' development of social interaction skills and learning process. In sum, the proposed method has been shown to enhance student motivation and reflection, which are critical to a productive, collaborative learning environment that directly impacts academic retention and performance.

KEYWORDS
Linear algebra; peer instruction; seminar strategy; SEEQ

1. INTRODUCTION

The linear algebra class is one of the first courses in which engineering students are challenged with numerous new definitions, theorems, and applications. Research indicates that a factor contributing to students' lack of understanding of linear algebra is the didactic pedagogical method used in the classroom [17]. The traditional approach—focusing on transmitting information to a group of passive learners, is less effective in teaching linear algebra [10]. Shifting the focus from transferring knowledge to creating and constructing it would benefit instructors and students alike [14].

This study proposes an alternative, blended pedagogical strategy for teaching linear algebra with appropriate evaluative tools. The approach incorporates peer instruction (PI), as developed by Mazur [24], and a seminar strategy to enhance understanding of mathematical concepts, stimulate investigative and creative interest, and promote the practical application of the studied concepts. Furthermore, the

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didactic engineering (DE) research methodology developed by Artigue [1] supports observation, organization, and analysis during class preparation. There is evidence that DE can contribute to developing didactical sequences to attain desired learning outcomes [11,13].

To assess the effectiveness of the proposed method, students' performance on exams and their evaluation of teaching quality were analyzed, and a diagnostic test was conducted at the onset of the study to identify any prior differences. Also, the Student Evaluation of Educational Quality (SEEQ) questionnaire, developed by Marsh [23], was administered to students in the course's final third. Extensive literature has established the SEEQ questionnaire as an effective tool for assessing teaching effectiveness [2,7,30,37].

The study contrasts the results between the experimental and control groups. To this end, this study compared the results of two exams and a SEEQ questionnaire from a class taught using the proposed approach with those of two classes using the traditional lecture-based teaching method.

PI methodologies have been extensively used in STEM courses [5,12,28,33] and in several mathematical disciplines [21,26]. While PI has been implemented in linear algebra courses [3,6,20], little is known about its effectiveness.

This study investigates the effectiveness of an alternative blended teaching methodology based on well-established active learning methods. The results were analyzed using validated evaluation instruments. More specifically, this study seeks to answer the following question: to what extent does the proposed active learning strategy improve student performance relative to traditional methods in a linear algebra course?

This study is organized into four sections: The first describes the implementation of the proposed and traditional methods in the classroom and the instruments used to evaluate their effectiveness. The second section provides data from the diagnostic test, exams, and SEEQ questionnaire, while the next discusses their implications for teaching effectiveness. The final section reflects on the findings of the study and notes further dimensions to be explored.

2. METHODS

The proposed mixed pedagogical strategy was implemented in the second semester of 2014 in the chemical engineering program of Brazil's Federal University of Ceará. Linear algebra is generally taught to first-year engineering and mathematics students as a 64-hour course, twice a week for two hours per class. While the basic textbook used in this course is Linear Algebra and its Applications by Gilbert Strang [32], instructors have the freedom to add additional resources.

The lectures covered the same sequence as the course outline, that is, matrices, determinants, linear equations, vectors, line and plane equations, vector spaces, basis, linear transformation, orthogonality, eigenvalue and eigenvector, and diagonalization. Among the 31 classes comprising the entire course, 16 implemented PI methodology, three used seminar strategy, and the remaining classes were divided into a review, exercises, quizzes, and tests. Of the 75 students (49 male and 26 female) enrolled in the course, 64 were for the first time. Their ages ranged from 17 to 21 years.

2.1. Peer Instruction Method

Consistent with the PI teaching method, multiple-choice conceptual questions are posed by the instructor for students to consider, address, and discuss with their peers in class. Student responses can be made through a polling system, using a display of hands, flashcards, or electronic devices connected to the instructor's computer. Subsequently, the instructor can provide explications as needed. The questions are intended to stimulate students' reflection and creativity and enhance their logical and abstract reasoning. Additionally, this discussion period seeks to encourage active student participation, debate, and cooperation [24].

A PI class organizes its content using key concepts. For each, a brief lecture was followed by a conceptual question. Students were encouraged to reflect on each individual. After two minutes, the first poll was taken, and the instructor recorded the responses. According to Mazur [24], an ideal correct response rate between 30% and 70% indicates that the question is sufficiently challenging, but not so difficult as to preclude meaningful discussion. For questions meeting Mazur's criterion, the instructor invited students to discuss their answers and attempt to persuade their peers of their suitability. After two to five minutes of discussion, a second poll was taken and, after recording the distribution of responses, the instructor provided feedback and explained the correct response. If the correct response rate falls below 30%, the instructor clarifies the concept and poses another conceptual question. Conversely, if the initial poll exceeds 70%, the instructor moves on to the next concept without peer discussion or a second poll.

Following the PI practices of the Just-in-Time Teaching method [25], students are provided material on the content of the next lesson for careful reading with conceptual questions to be answered by the students, thus exercising their reasoning and facilitating a rich and productive class discussion. The instructor may pose two questions: one based on challenging aspects of the assigned reading, and another designed to motivate students to think about its content, for example, "Did you find anything difficult or confusing in the reading? What did you find the most interesting?"

The just-in-time method anticipated that completed reading assignments would be delivered to the instructor promptly to enable adjustment of the teaching plan, an assumption that was not practical. However, considering the local reality, the instructor would not have enough time to analyze the reading assignments to adjust the plan for the next lesson; then, two phases of DE methodology [1] were used. DE was inspired by the work of the engineer in the meaning of the conception, planning, and execution of a project. It is divided into four phases: preliminary analysis that studies the historical, epistemological, cognitive, and didactic aspects associated with some content; a priori analysis focuses on the preparation of didactic

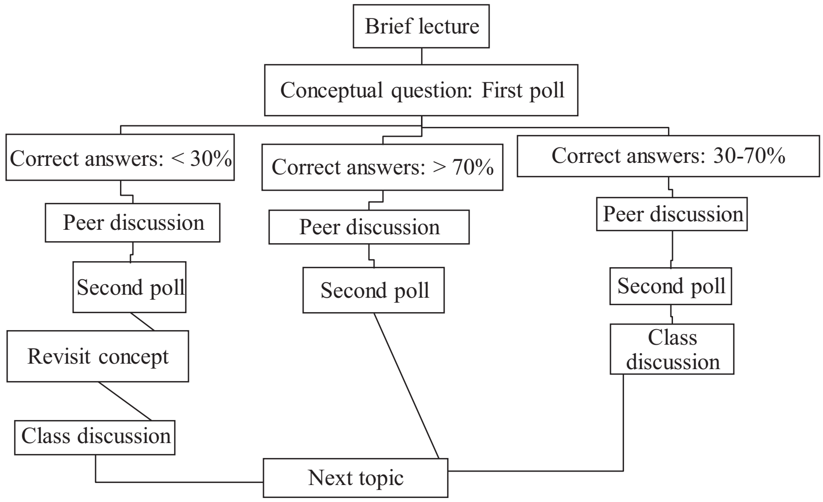

Figure 1. PI class instructional sequence.

sequences considering students' previous knowledge; experimentation, which is the implementation of the didactic sequences; and a posteriori analysis—validating the hypotheses previously raised in the a priori analysis phase. Consistent with the a priori analysis phase of DE, the instructor collects the reading assignments at the onset of class, and the lessons are prepared by anticipating questions that could arise from reading assignments in light of the instructor's experience and expertise and the relevant literature. Following the class, the questions are analyzed and compared with the inferred predictions, following DE's a posteriori analysis phase. Finally, if necessary, the instructor will discuss concepts not included in the initial forecast in the next class, adjusting the teaching plan to meet student needs.

In the study's implementation of the proposed method, students were provided a set of four cards at the beginning of the class to use in polling. After a brief discussion of the reading assignments, a series of presentations, each focused on a critical point, followed by a related conceptual question were conducted. Students were encouraged to reflect on their responses independently for two to four minutes. Subsequently, the first round of polling was conducted, and the responses were cataloged and analyzed. Regardless of the correct response rate, students were encouraged to discuss their responses with their classmates in small groups, usually comprising students with diverse answers. After approximately four minutes, the instructor polled the class a second time and explained the correct answer before presenting the next topic. For a graphic depiction of the PI sequence, see Figure 1. For further details regarding its components, see [35].

The concepts are explained using simple to complex examples to enable students to surmount initial theoretical challenges. Accordingly, class content flows from concrete to abstract, evolving, when possible, from geometric visualizations and physical interpretations to more abstract concepts [14]. Then, at the end of

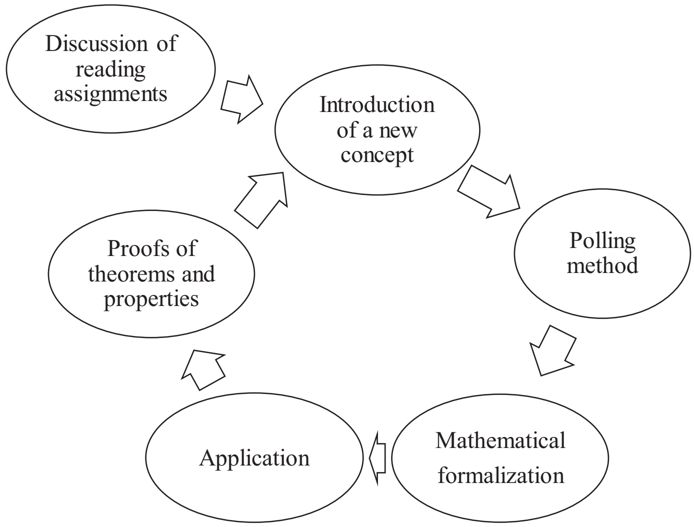

Figure 2. Flow chart of a linear algebra lecture.

each lecture, when appropriate, the instructor constructed proofs of the main theorems and properties based on class discussions. The sequence depicted in Figure 2 is frequently followed to manage class time more efficiently and to optimize lectures.

2.2. Seminar Strategy

The seminar was conducted close to the semester to promote learning in a more meaningful way. The seminar's strategy is characterized by the study and research of a theme—enabling a systematic process of reading, analyzing, interpreting texts, and data to formulate a research problem, a hypothesis, and investigation [22]. This pedagogical strategy enables researchers to develop and create conditions for students to learn to elaborate mathematical models and apply theory to practice, thus stimulating their motivation [4,8].

Typically, the seminar comprised an oral presentation and a written report by the students, usually divided into teams with pre-established content. In this study, the class was divided into 13 teams comprising four to six students. Teams were free to select their topics for examination under the supervision of the instructor. The themes dealt with the applicability of linear algebra, such as changes in coordinates in color systems; eigenvalues and eigenvectors: Google page rank; linear transformation: application involving rotation and reflection; application of the spectral theorem in the decryption of messages as matrices, linear algebra applied to computer science, eigenvalue, and eigenvector applications: Markov chains, application of eigenvalues and eigenvectors in search engines, and differential equations.

The activities' objectives, organization, elaboration, and evaluation criteria were set forth, and the teams were guided throughout their projects' development process and encouraged to engage in more critical thinking.

Oral presentations were evaluated using the following criteria: individual presentation, organization, content knowledge, timeliness, and audio/visual quality. Written reports were evaluated according to organization, methodology, and mathematics accuracy. Each group was given 20 minutes of oral presentation and 5 minutes of discussion.

2.3. Learning Evaluation Methods

The evaluative process was based on integrating the results of the diagnostic and formative evaluations to identify students' prior difficulties and determine their causes during learning activities. Reading assignments, class participation, quizzes, two tests, and the seminar were used in an ongoing assessment to identify class content and activities and guide extracurricular research. All assignments had a maximum score of 10 points.

In evaluating a student's completion of the reading assignments, the responses were analyzed and consistency with the correct answer was required to be awarded a maximum of 10 points for each. The reading assignments and quizzes were intended to motivate continuous study throughout the semester and to assist the instructor in identifying the students' principal challenges.

Two exams comprised conceptual questions based on reading assignments and conceptual questions discussed in class (60%), and computational questions based on textbook problems (40%) were conducted during the semester. Equal distribution of points across questions was applied to equate qualitative and quantitative reasoning and normalize scoring. One conceptual question was open, while the other comprised several statements that students marked true or false with justification. The first exam was composed of basic concepts of the initial phase of the linear algebra course and the notion of vector space, while the second was composed of vector space, generator, and base and linear transformation.

Students' overall assessment comprised two partial evaluations and a seminar. Each partial evaluation accounted for a maximum score of 10 points and was composed of 10% of the reading assignments, 90% of the test score, and an extra 10% for quizzes.

Seminar grade was also normalized to a maximum of 10 points distributed according to a preset rubric: written work (3 pts), oral presentation (5 pts), and attendance/participation on presentation days (2 points).

Students' final score was then determined by:

(i) Two Partial Evaluations (reading assignments + tests + quizzes): 40% each
(ii) Seminar: 20%

On the first day of class, a diagnostic test to identify the level of students' prior knowledge of basic concepts related to linear algebra was conducted, whose individual results were not provided to the students. The same test was subsequently conducted after the contents had been taught.

Table 1. Analyzed SEEQ factors.

Abbreviation*	Factor		Students' opinions of:
Lrn	Learning (4 items)		Challenge and learning value of course
Enth	Enthusiasm (4 items)		Instructor's dynamism in class
Org	Organization (4 items)		Instructor's organization and preparation for class
GrpInt	Group Interaction (4 items)		Student engagement in class
IndRap	Individual Rapport (4 items)		Relationship between instructor and students
Brdh	Breadth (4 items)		Presentation of concepts' backgrounds, different viewpoints, and current developments
Exam	Examinations (3 items)		Instructor's assessment and grading methods
Asgn	Assignments (2 items)		Appreciation of readings, homework, etc.
O_C/I	Overall	Course (1 item) O_Cour	Relative course difficulty.
		Instructor (1 item) O_Ins	Relative instructor difficulty.
O_Char	Overall Student and Course Characteristics (4 items)		Relative course difficulty, class pace, and time required for outside class activity.

*N.B.: Factor abbreviations used in all subsequent tables.

2.4. Instruction Evaluation Methods

2.4.1. Student Perception

To assess student perception of the proposed method and its contribution to learning, an adapted version of the SEEQ questionnaire comprising 35 items in ten dimensions was administered before the seminar (see Table 1). It was used a Portuguese version of the original SEEQ questionnaire, validated in Brazil via confirmatory factorial analysis [30,31] (see SEEQ adapted in Appendix). The last five questions concerning administrative issues were removed from the instrument, as they were outside the scope of our current research. The items were measured using a five-point Likert scale, ranging from 1 (very little) to 5 (great). For a description of the factors, see Marsh [23].

2.4.2. Comparison Between Experimental and Control Groups

To assess the efficiency of the proposed method, a comparative analysis between the experimental group (EG), students using the proposed method, and two control groups (CG1 and CG2) using the traditional method, was performed.

The study was conducted in the same semester at the same university in two departments with the same linear algebra syllabus. The EG was taught in the chemical engineering department, while both control groups were taught in the Department of Energy and Environmental Engineering of the Universidade Federal do Ceará. While CG classes were taught by another instructor, EG and CG instructors met periodically to coordinate their courses and develop similar assignments and scoring rubrics.

To compare the relative effectiveness of the two methods, the same diagnostic test, two similar tests (partial evaluations), and the SEEQ questionnaire, as described above, were conducted in the same period.

2.4.3. Control Groups

The 63 subjects enrolled in the morning traditional lecture class (CG1) comprised 36 males and 27 females, while the 47 enrolled in the evening counterpart (CG2)

Table 2. Percentage of students by gender per class.

Class	*N* (number of enrolled students)	Male (%)	Female (%)
EG	75	65.33%	34.67%
CG1	63	57.15%	42.85%
CG2	47	61.7%	38.3%

comprised 29 males and 18 females. The CG classes consisted primarily of lectures and problem solutions by the instructor, with occasional encouragement of student participation. The instructor provided lecture notes and homework assignments and conducted office hours and review sessions for exams. Table 2 shows the percentage by gender of the enrolled students in each class.

3. RESULTS AND DISCUSSIONS

3.1. Results of Proposed Method

To determine the effect of the proposed method, a comparison was made between student performance in the conceptual questions in the polling and pre-and post-class tests. The completion rate of students in reading assignments, their performance in quizzes, their participation and performance in tests and seminars, and their assessment of the teaching method were analyzed.

3.1.1. Pre-and Post-class Tests

The pre-class test was conducted on the first day of class, with 60 (85%) of the enrolled students. It comprised ten assertive questions covering prior knowledge of basic concepts needed to understand linear algebra, such as matrices, systems of linear equations, determinants, notions of vectors, and equations of line and plane. The questions were carefully selected by both the EG and CG instructors—all members of the engineering education research group—to ensure that the expected learning outcomes were met. The same test was conducted after the classes were completed. For this analysis, only the results of 58 students who took both tests were evaluated.

The pre-and post-class tests recorded means of 3.29 and 6.63 (out of 10) respectively. Student's t-test results were used to determine whether there was a significant difference in performance [38]. The results ($t = 13.380$; $p = 0.000$) confirmed that the increase was statistically significant. For further details on the test analyses, see [34].

The results of the pre-class test reveal a low level of knowledge of basic concepts essential to understanding linear algebra with which engineering students enter universities. The results of the post-test, however, demonstrate how the proposed method can contribute significantly to advancing the understanding of these critical concepts. As an example, Table 3 presents the most critical questions in the pre-test. Questions on vectors and line equations achieved 18% of correct responses in the pre-test and 38% and 33%, respectively, in the post-tests.

Table 3. Pre-test applied in a statement format.

Concepts	Statement
Vector	The length of every vector is a positive number.
Line	The intersection between two planes is a line.

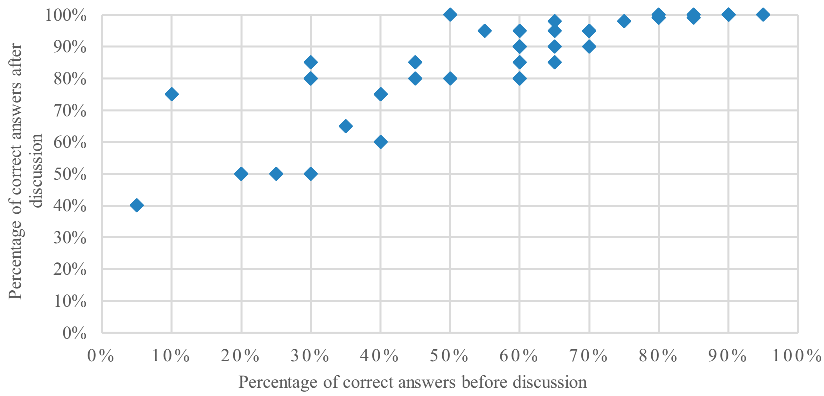

Figure 3. Percentage of correct answers before and after discussion.

3.1.2. Conceptual Questions

Data collected during the observations were descriptive and recorded in writing. An approximation was made of the percentage of correct answers in the first and second polls. During the semester, 43 conceptual questions are discussed. The first 20 involved basic concepts necessary for understanding linear algebra, as previously described, and the remaining 23 covered new content introduced in the course. While conceptual questions were collected from textbooks and related websites, they all passed through careful re-elaboration. Such an endeavor was necessary to isolate specific concepts and desired properties or skills for discussion in class [3,15,18,19,27].

Figure 3 depicts the percentage of correct answers before and after the discussion. Each point corresponds to a single conceptual question. For most questions, the results showed a significant increase in correct answers in the second poll.

In the first poll, it can be observed that 12% of the conceptual questions corresponded to less than 30% of correct answers and that 23% surpassed 70% of them. A margin of 30%–70% of correct answers was reached in 65% of the questions, a condition deemed ideal according to the principles of PI.

A review of responses indicated that questions involving vector space generated considerable doubts and discussions among students. Table 4 exemplifies one of these questions. The initial polling generated a correct response rate of only 30%. The question prompted a particularly stimulating discussion in the class. During the second voting process, more than 80% of the students opted for the correct answer. Contrastingly, questions involving matrices and linear equations featured a correct response rate of above 70% in the first poll. For example, for the question shown in

Table 4. Conceptual question – basis. (correct answer: b).

Let E be a vector space with dimension 17 and S a subset of n elements. Among the following properties, which one is not always true?
(a) S spans E and $n = 17 \Rightarrow S$ is a basis for E
(b) S contains a basis for $E \Rightarrow S$ is linearly independent
(c) S is a basis for $E \Rightarrow S$ is linearly independent and S spans E
(d) S is linearly independent $\Rightarrow$ There is a base for E that contains S

Table 5. Conceptual question – linear systems (correct answer: a).

Consider a system of n linear equations, n variables with n pivot positions in the reduced echelon form of its matrix, then
(a) the system has a unique solution
(b) the system doesn't have solution
(c) the system has n solutions
(d) the system has infinitely many solutions

Table 6. Conceptual question – linear independence and dependence (correct answer: b).

Let E be a vector space and S a non-empty set. If S is linearly dependent, then			
I. 0 is a linear combination of elements of S.			
II. any vector of E is a linear combination of elements of S.			
III. S contains a vector that is a linear combination of the others.			
IV. S contains a vector and its negative.			
Which the following sequence is correct?			
(a) V, F,V,V	(b) V,F,V,F	(c) F,V,F,V	(d) F,F,V,F

Table 7. Conceptual question – matrix properties. (correct answer: d).

If a square matrix A is such that $A^2 - 4A^3 = 0$, then
(a) $A^2(1 - 4A) = 0$
(b) $A = 0$ or $A - 4A^2$
(c) $\frac{A^2}{A^3} = 4$
(d) none of the above.

Table 5, 90% of students got it right in the first pool, and no further explanation was required.

A question regarding the concept of linear dependence (Table 6) was the most confusing one, measured by the balance between the choices in the first poll (25% for each item). During peer discussions, a warm debate on the alternatives was observed, yielding a balanced 50% for each alternative (b) and (d) in the second poll.

The highest percentage increase was in the matrix multiplication property question (Table 7). Its initial polling generated a correct response rate of just 10%, and there was an increment of 65 percentage points in the second poll. An analysis of each potential response was conducted. Most students did not distinguish between the rules governing matrix multiplication and those applicable to the arithmetic multiplication of real numbers. The question prompted a stimulating discussion among the class, intensifying students' buy-in and motivation for active, collaborative participation in the course.

Table 8. Reading assignment with very high failure rate – determinants.

Given a square matrix A and λ a scalar. Show that $\det(A - \lambda I) = 0$ if and only if $Ax = \lambda x$ for some $x \neq 0$.

The integration and collaboration among students showed their commitment to resolving the proposed questions. The significant increase in their success following the discussion demonstrates the PI value.

3.1.3. Reading Assignments

Fourteen readings were assigned during the semester and selected from a variety of textbooks [15,18,19,27,29]. On average, 80% of the students completed the seven reading assignments related to the first evaluation, and 65% completed the seven assignments related to the second.

When asked what they found difficult or confusing, several students asserted that the reading material was inadequate to respond to the assignments and that additional sources were required to understand their content. In this regard, it should be noted that each reading assignment was accompanied by complementary bibliographical references. To address this matter, the instructor explained that the reading material served as a study guide. Following this clarification, students began to collaborate effectively, identifying the points they deemed most complex and their evaluations thereof.

The reading assignment with the highest failure rate was a math proof testing ability, as presented in Table 8. An overwhelming majority of students (90%) failed to find the correct solution. Students showed difficulty in discerning matrix multiplication properties from real number multiplication properties. As such, the instructor provided another explanation of the concept, using a different approach, and then the question was resubmitted as a reading assignment. In the second round, 75% of the students answered the questions correctly.

Generally, students demonstrated difficulties in solving preliminary questions, principally those regarding the course's more advanced content. Among potential contributing factors, a lack of ability to solve conceptual problems that require logical reasoning is particularly relevant.

Reading assignments provide useful information about students' confusion, feedback that demonstrates that students are concerned about solving challenges and will take advantage of a means to communicate their difficulties to the instructor in an efficient and organized manner.

3.1.4. Quizzes

Six quizzes were administered during the semester; the first three added at most one point to the total of the first partial evaluation and the latter three to the second. As these were extra points, participation in the quizzes was not mandatory. Nevertheless, nearly 85% of the students took all quizzes in the first partial evaluation, attaining an average of 0.58 additional points, while 77% of all quizzes in the second, adding an average of 0.75 points. This indicates students' commitment

Table 9. Example of a typical textbook question and its version in exam.

Textbook question	Exam question
Determine whether the following system of equations is consistent. If so, show its solution. $$\begin{cases} x+y+2z=8 \\ -x-2y+3z=1 \\ 3x-7y+4z=10 \end{cases}$$	Consider the following linear system of equations. Which the following options would be true? Select all that apply. Justify your answer. $$\begin{cases} x+y+2z=8 \\ -x-2y+3z=1 \\ 3x-7y+4z=10 \end{cases}$$ (a) The system has no solution. (b) $x=2, y=2$, and $z=2$ is a solution of the system. (c) The system has exactly one solution. (d) If $z=1$, then $x=2$ and $y=4$.

and demonstrates their proactive stance. The results further show that quizzes, like reading assignments, stimulate students to continue studying while enabling the instructor to identify the most difficult challenges facing students.

3.1.5. Exams

A total of 68 students (90%) took part in the first exam (E1), attaining an average score of 6.56 out of 10. In the second exam (E2), 65 students (87%) participated, with an average of 6.97 out of 10. Half of the students taking E1 and 57% of those taking E2 scored higher than the means of the respective partial exams, 31% and 42% of students achieved excellent or very good excellent grades (8–10 pts) in E1 and E2, respectively.

The questions featured in the midterm exams were carefully modified from textbook ones [3,15,18,19,27], keeping the same learning objectives, but presented rather differently to avoid solutions by procedure memorization. Table 9 shows an example of how exam questions were reformulated based on standard textbook problems.

Analysis of the tests indicates that the greatest difficulties encountered by students involve conceptual questions, particularly those requiring greater capacity for logical and abstract reasoning. It should be noted that these questions were those that received the most attention in class and that these difficulties were more prevalent in the first evaluation.

For example, in question 1 of the first exam (Table 10), students found it difficult to explain the items correctly. A common misconception was to argue only in particular cases when $n = 2$ or 3. Most students were unsuccessful in obtaining an abstract argument applicable to general cases. This example demonstrated the deficiency that students had with mathematical reasoning and, to minimize this gap, the instructor started to explore more proofs of theorems and properties during class. Subsequently, it was observed an improvement in students' logical reasoning skills in reading assignments and tests.

The results obtained in both tests reflect good performance in both the absolute and comparative terms. Thus, it is fair to conclude that the proposed method has a positive effect on enhancing the knowledge inherent to linear algebra.

Table 10. Question E1 – Hardest exam 1 question according to students.

Question 1 – Let $Ax = b$ a system with n linear equations and n variables. Suppose that x_1 and x_2 are both solutions of the system and $x_1 \neq x_2$.
(a) How many solutions do the system have? Justify your answer.
(b) Is the matrix A non-singular? Justify your answer.

3.1.6. Seminar Strategy

The seminar group comprised 13 teams of four to six students. Their themes addressed various applications of linear algebra. The seminar enabled evaluation of the performance of both the team and its members regarding oral presentation, debate, and written work, recording the main flaws. Besides, the assessment provides students with relevant questions and offers an extended discussion.

During oral presentations, notes were compiled for each participant, evaluating their presentation, clarity and objectivity, theme domain, adequate use of time, and didactic resources. Written works were evaluated in terms of synthetic, clear, and sequential information, linguistic adequacy, and their objectives, methodologies, results, and conclusions. Scrutiny differentiated active team members from those with little or no participation. In this regard, the instructor instigated and analyzed students' participation in the discussion section. Two classes were designated to the seminar in which students received 0.5 points for attendance and 0.5 for participation in each class. Records made during the process provided critical feedback to all students in the class. The quality of their slides and written works reflects students' commitment, resourcefulness, and care, and their interest in classmates' presentations significantly stimulated interactive engagement among groups.

Based on study data, the seminar strategy is an effective way to show students how research can contribute to learning while promoting reflection, questioning, argumentation, communication, discussion, and exploration of the relationship between theory and practice. Thus, it can be seen that the seminar strategy supports meaningful learning and contributes to students' maturation via their critical, proactive formation, which is indispensable to the academic training of engineers [4,8].

3.1.7. Student Performance Analysis

Analysis of students' grade performance found that 78% of the 75 students enrolled passed, 13% failed, and only 9% dropped out. After normalizing the final scores to the 100 scales, the average was 72.4 ($SD = 2.51$), with 60% of students excelling the average and 27% scoring greater than or equal to 90, indications that linear algebra concepts were effectively assimilated and that the proposed method could reduce the high dropout rate in linear algebra engineering courses.

3.1.8. Student Perception

To investigate students' satisfaction with the methodological process and its contribution to the learning process, the SEEQ questionnaire was administered to 59 students before the seminar.

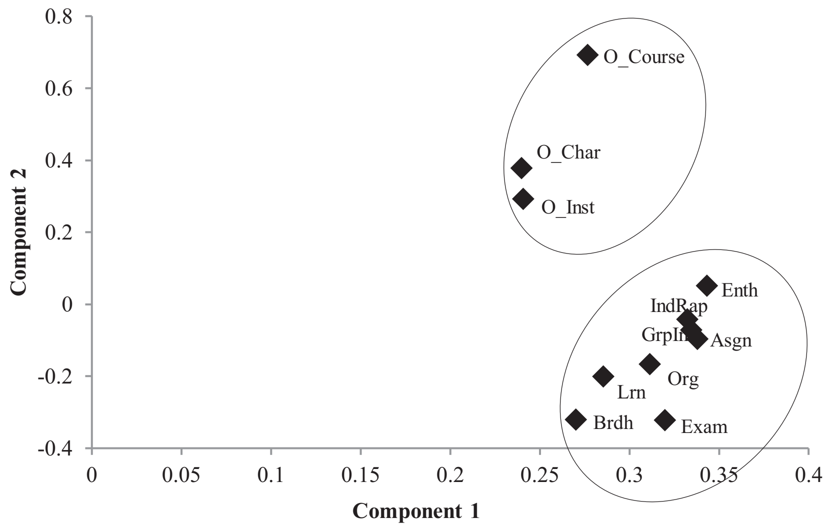

Figure 4. Principal Component Analysis score plot for experimental group.

Cronbach's alpha was used to estimate the reliability of the responses for each factor [9]. The results revealed that 90% of the factors yielded a high level of internal consistency α ($\alpha > 0.7$), indicating that their responses were reliable and admissible.

Principal component analysis (PCA) was used to identify the interrelationships among factors based on the mean score of student responses to each factor [16]. PCA identified two distinct clusters: one related to teaching and learning (Figure 4, component 1) and the other related to overall aspects of the course and the instructor (Figure 4, component 2). Together, they account for 98.33% of the variance in the original data matrix. Examination of these clusters yields a more precise understanding of the factors that influence students' perceptions of the designed teaching method.

As depicted in component 1, learning, enthusiasm, organization, group interaction, individual rapport, breadth, examinations, and assignments are intrinsically interrelated. These factors share a common approach to teaching and learning, specifically regarding the connection between the instructor's behavior, evaluation methods, and learning process.

As component 2 illustrates, the course, instructor, and characteristic factors have similar loading factors. These factors have a strong relation to such general characteristics as course requirements, efforts, and student perception of their instructors.

The degree of satisfaction for all factors exceeded 3.00 on a five-point Likert scale. Additionally, six (54%) of the 11 factors analyzed (enthusiasm, assignments, group interaction, individual rapport, exams, and organization) had an average score exceeding 4.00, demonstrating a high level of student satisfaction with the proposed method and assessment used by the instructor, with most of them very positively (see Figure 5). Indeed, more than 75% of the students affirmed that the course had a

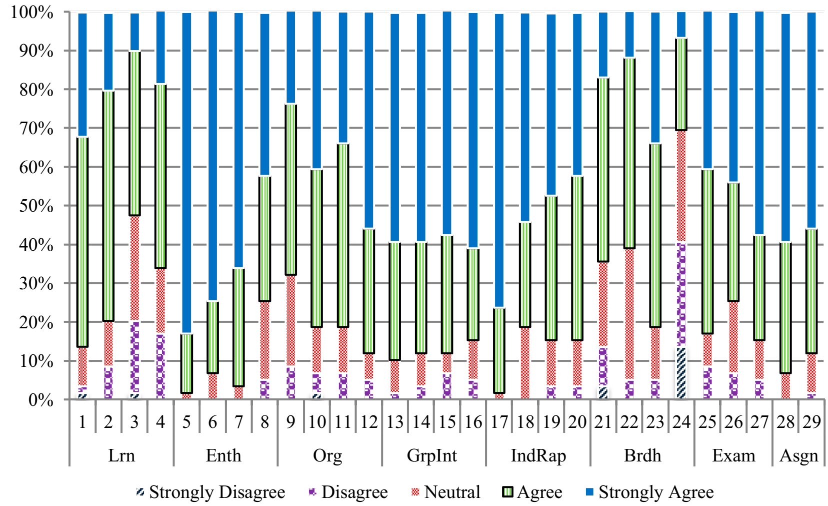

Figure 5. Student opinion by factor.

positive impact on their learning process and that the evaluative methods were fair and appropriate.

Students strongly agreed that the instructor was enthusiastic in conducting the course, with 83% rating this factor 5 on the Likert scale, and 73% considered the instructor dynamic and energetic in driving the class. Nearly 90% of the students confirmed that they were encouraged to participate in classroom discussions and to share their ideas and questions, rating this factor 4 or 5, and 88% thought that the reading assignments contributed to their appreciation and understanding of the content.

The rate of negative responses ("strongly disagree" and "disagree") was relatively low, with its highest index for question 24 (coverage), in which 14% of respondents disagreed strongly, 27% disagreed, and 29% were neutral when asked if the instructor adequately commented on current research in linear algebra.

The course was deemed reasonable by 39% and difficult by 44% of the students when compared to other courses, and the instructor fair by 64% compared to other instructors. The course was regarded as moderate in level of difficulty and workload by 47% and 46% of the students, respectively. However, 49% of the participants found it was given at a rapid pace. Most students (68%) reported spending two to five hours per week studying linear algebra.

The results demonstrate the significant acceptance of the proposed method and the assessment used by the instructor. Thus, the consistency and connection between the sequences that comprise the method become evident, and student perception provides additional support for its efficiency.

3.2. Results of the Comparison Between Experimental and Control Groups

Students' diagnostic tests, exam performance, and the SEEQ questionnaire were reviewed to compare the effectiveness of the proposed method using the EG with

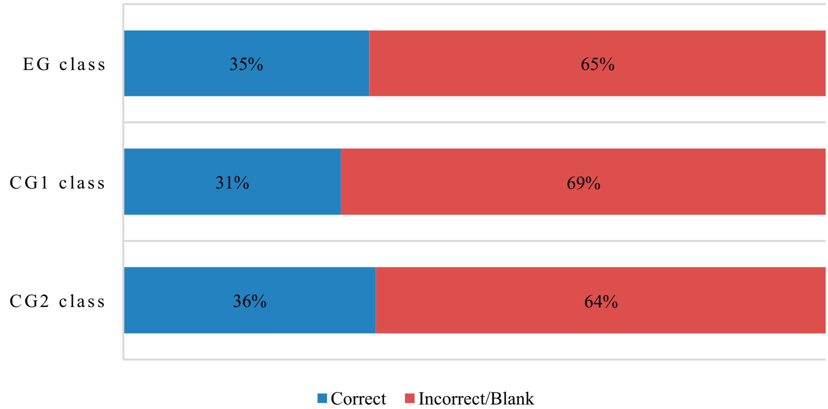

Figure 6. Aggregate results of diagnostic test per class, combining incorrect and blank responses.

that of the traditional lecture method, used with the two control groups (see [36], for details).

3.2.1. Diagnostic Test

The pre-class diagnostic test was administered on the first day of the class in both groups. In the EG class, 85% of the students took the test, and in the control classes, the corresponding rates for the CG1 and CG2 classes were 85% and 95%, respectively. Figure 6 depicts the aggregate results when incorrect and blank responses are included. The findings showed no significant differences, indicating that the groups had similar backgrounds and abilities before the course. As previously noted, a notable finding of this study is the significant deficiency in first-year students' knowledge of basic mathematical concepts essential to understanding linear algebra, as confirmed by the pre-class diagnostic test results. The post-class test was not administered to the control group; therefore, no analysis was conducted.

3.2.2. Exams

For each exam, a t-test was used to determine whether there was a statistical difference between the EG and CG1 and the EG and CG2 means. As Table 11 shows, students in the class using the proposed method performed better in the two exams than those in the classes using the traditional method. In the first exam, EG students (mean $m = 6.56$) performed better than the CG students (CG1: 5.67; p-value $p < 0.028$, CG2: $m = 5.06$; $p < 0.001$), and in the second exam, they ($m = 6.97$) attained significantly higher scores than the CG2 students ($m = 5.3$; $p < 0.0021$). Although the EG mean score was greater than that of CG1, no significant difference was observed between them. In summary, the exam findings indicate that the proposed method enhances students' mastery of basic linear algebra concepts more effectively than the traditional method.

Figure 7 shows that the median score of the EG group in exam 1 was 66%—considerably higher than the CG1 and CG2 classes, which achieved 57% and 55% of the median, respectively. Also, given the much longer "whiskers" for control groups, it

Table 11. Exam performance per class.

Class	Exam 1			Exam 2		
	N (% of enrolled students)	Mean 95% CI	*p*-value	*N* (% of enrolled students)	Mean 95% CI	*p*-value
EG	68 (90%)	6.56 ± 0.54		65 (87%)	6.97 ± 0.62	
CG1	59 (93%)	5.67 ± 0.72	0.028*	55 (87%)	6.36 ± 0.82	0.12
CG2	40 (85%)	5.06 ± 0.79	0.001*	34 (72%)	5.3 ± 1.01	0.002**

*Statistically significant results, $p < 0.05$.

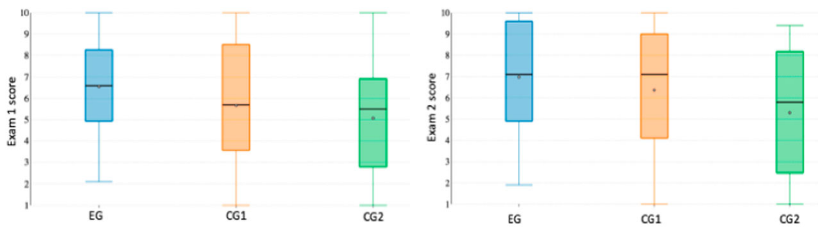

Figure 7. Distribution of scores on the exams per class represented as a boxplot graph.

can be interpreted that those students vary more widely in the scores than the EG students. For exam 2, 50% of both the EG and CG1 classes scored 71% and CG2 class scored 58% or higher. This indicated that half of the EG and CG1 students performed similarly well and better than the CG2 students. It is worth mentioning that 25% (16 out of 65) of the EG students scored 96% or higher.

3.2.3. SEEQ Questionnaire

The SEEQ questionnaire was administered to the final third of the course. As SEEQ data from the EG have been presented above, this subsection focuses on the results from the CG classes. As both were taught by the same instructor, their students completed the questionnaires simultaneously. The participation rates for the EG and CG students were 86% and 60%, respectively.

Cronbach's alpha α was calculated to examine the internal consistency of the SEEQ responses for each factor. Reliability varied between 0.51 and 0.78 for CG classes with half the factors presenting high internal consistency. Although some factors showed low internal consistency, the reliability of the overall factors was satisfactory.

PCA was used to assess the relationships among the SEEQ questionnaire factors in the control group. Figure 8 depicts two principal components, one related to teaching and learning (component 1), and the other related to overall aspects of the course and the instructor (component 2). Together, they accounted for 98.33% of the total variance. The findings indicate that the significant factors for these components were identical to those for the EG. Hence, in both groups, PCA succeeded in extracting the same principal components.

An independent sample t-test was conducted to compare SEEQ factor scores. The results indicated a statistically significant difference between the EG and CG

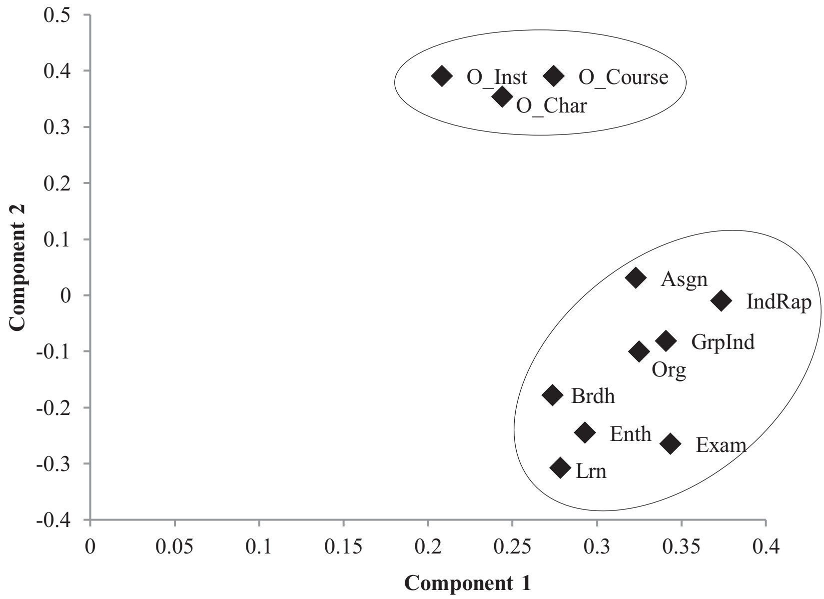

Figure 8. Principal Component Analysis score plot for control group.

classes in mean scores of learning ($p = 0.044$; Cohen's effect size $d = 1.53$), enthusiasm ($p = 0.006$, $d = 2.5$), group interaction ($p = 0.014$, $d = 2.6$), and assignment ($p = 0.038$, $d = 3.07$) factors. These findings indicate that student motivation for learning was greater in the EG than in the control group.

Figure 9 depicts the SEEQ results for the first eight factors per class for "strongly agree" and "agree" responses (see Appendix for the statement's description). The EG class exceeded the CG class in over 75% of the questions. For the remaining 25%, in which CG classes' affirmations surpassed those of the EG class, it should be noted that 43% involved the individual rapport factor, 28% the organization factor, and 28% the examination factor. This suggests that the CG instructor was perceived by students as friendly, accessible, and well-organized and that the method of evaluating student work was deemed fair and appropriate.

In terms of the overall factors (O_C/I, O_Char), there were no statistically significant differences between the classes. Some 39% of EG and 43% of CG students deemed linear algebra a moderate course as compared with others. From the same perspective, 64% of EG and 47% of CG students found their instructor fair. Half of the students in the EG and CG classes considered their course moderate regarding difficulty and workload, and about 45% found it fast paced. Over 68% of EG students, as opposed to 40% of CG students, spent two to five hours a week studying linear algebra outside the classroom.

The results indicate that the mixed method creates a more dynamic learning environment for students than the traditional method, contributing to students' development of social interaction skills and learning processes. Furthermore, the mixed method's facilitation of enhanced comprehension of course content is likely

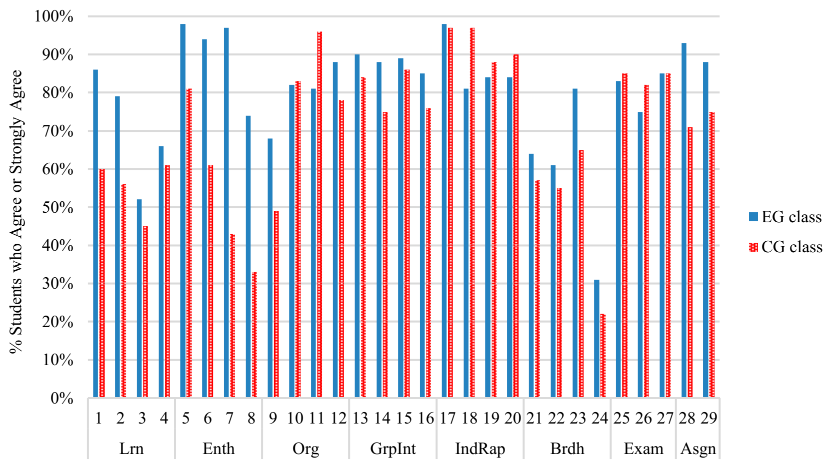

Figure 9. Student opinion by factor for "strongly agree" and "agree" responses per class.

to contribute to better academic performance, as evidenced by the EG students' exam grades.

Such results make a strong case for the positive effect of the proposed method on students' perception of their academic environment and their willingness to actively participate in learning.

3.3. Limitations of the Research

It should be noted that the classes were taught by different instructors in different departments, and the two groups of students had slightly different majors. While instructors held regular meetings to discuss the progress of the courses, to develop similar assignments and scoring rubrics, the same conditions for groups could not be controlled. Variables such as the number of students enrolled in each class, students' genders, and students' class standing could not be controlled in this experiment. The authors recognize that these variables may have played a role in the results of this study, but in the end, they believe that their other evidence remains compelling. Recommendations for further research on this theme include similar experiments, considering the unmeasured variables. Bias is always difficult to avoid, although joint coordination of both classes may have helped minimize bias and thus produce reliable findings.

4. CONCLUSION

The blending of PI with a seminar strategy, supported by didactical engineering, and in conjunction with relevant evaluative methods, met its primary objectives. As documented in this study, the proposed method offers significant advantages over traditional methods in numerous areas. It proved effective in enhancing students' understanding of abstract concepts integral to learning linear algebra and facilitating their practical application, thereby reducing the rates of academic failure and dropout. As students mastered these concepts, they assumed greater responsibility for their learning, increasing their active participation in the learning environment, a critical component of academic achievement.

The integration of this innovative method in linear algebra courses enriches the learning experience for students and instructors alike.

The findings of this study, however, are not restricted to the boundaries of its particularities and suggest the potential application of the method not only to other subjects in the engineering and mathematics syllabus but to other disciplines as well. Further research along these lines merits consideration.

Appendix. The Student's Evaluation of Educational Quality (SEEQ) Instrument

Use the following to evaluate the first 29 statements: (Strongly Disagree, Disagree, Neutral, Agree, Strongly Agree)

LEARNING:

1 You found the course intellectually challenging and stimulating
2 You have learned something which you consider valuable
3 Your interest in the subject has increased as a consequence of this course
4 You have learned and understood the subject materials in this course

ENTHUSIASM:

5 Instructor was enthusiastic about teaching the course
6 Instructor was dynamic and energetic in conducting the course
7 Instructor enhanced presentations with the use of humor
8 Instructor's style of presentation held your interest during class

ORGANIZATION:

9 Instructor's explanations were clear
10 Course materials were well prepared and carefully explained
11 Proposed objectives agreed with those actually taught so you know where the course was going
12 Instructor gave lectures that facilitate taking notes

GROUP INTERACTION:

13 Students were encouraged to participate in class discussions
14 Students were invited to share their ideas and knowledge
15 Students were encouraged to ask questions and were given meaningful answers
16 Students were encouraged to express their own ideas and/or question the instructor

INDIVIDUAL RAPPORT:

17 Instructor was friendly toward individual students

18 Instructor made students feel welcome in seeking help/advice in or outside of class
19 Instructor had a genuine interest in individual students
20 Instructor was adequately accessible to students during office hours or after class

BREADTH:

21 Instructor contrasted the implications of various theories
22 Instructor presented the background or origin of ideas/concepts developed in class
23 Instructor presented points of view other than his/her own when appropriate
24 Instructor adequately discussed current developments in the field

EXAMINATIONS:

25 Feedback on examinations/graded materials was valuable
26 Methods of evaluating student work were fair and appropriate
27 Examinations/graded materials tested course content as emphasized by instructor

ASSIGNMENTS:

28 Required readings /texts were valuable
29 Readings, homework, etc., contributed to appreciation and understanding of subject

OVERALL:

30 Compared with other courses you have taken at your department, this course was (1-Very poor … 3-Average … 5-Very good)
31 Compared with other instructors you have had at your department, this instructor was (1-Very poor . .. 3-Average … 5-Very good)

STUDENT AND COURSE CHARACTERISTICS: (Leave blank if no response applies)

32 Course difficulty, relative to other courses, was (1-Very easy … 3-Medium … 5-Very hard)
33 Course workload, relative to other courses, was (1-Very light … 3-Medium … 5-Very heavy)
34 Course pace was (1-Too slow … 3-About right … 5-Too fast)
35 Hours per week required outside of class (1) 0–2; (2) 2–5; (3) 5–7; (4) 7–12; (5) Over 12.

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REFERENCES

[1] Artigue, M. 1990. Ingénierie didactique. *Recherches en Didactique des Mathématiques*. 9: 281–307. La Pensée Sauvage.
[2] Balam, E. M. and D. M. Shannon. 2010. Student ratings of college teaching: A comparison of faculty and their students. *Assessment & Evaluation in Higher Education*. 35(2): 209–221.

[3] Beites, P. D. and A. P. Nicolás. 2013. Peer Instruction in Linear Algebra. In *Proceedings of ICERI2013 Conference*, Seville, pp. 6629–6639.
[4] Berry, F. C., P. S. DiPiazza, and S. L. Sauer. 2003. The future of electrical and computer engineering education. *IEEE Transactions on Education*. 46(4): 467–476.
[5] Chen Y., W. Lin, S. Chang, and C. Liu. 2005. TIPS: a JiTT & PI pedagogical method with handheld computer as mediating tools. In *Proc. of IEEE Advanced Learning Technologies* (ICALT), pp. 844–845. Kaohsiung, Taiwan.
[6] Cline, K., H. Zullo, J. Duncan, A. Stewart, and M. Snipes. 2013. Creating discussions with classroom voting in linear algebra. *International Journal of Mathematical Education in Science and Technology*. 44(8): 1131–1142. doi:10.1080/0020739X.2012.742152
[7] Coffey, M. and G. Gibbs. 2001. The evaluation of the student evaluation of educational quality questionnaire (SEEQ) in UK higher education. *Assessment & Evaluation in Higher Education*. 26(1): 89–93.
[8] Crawley, E. F. 2002. Creating the CDIO SYLLABUS, a universal template for engineering education. *32nd ASEE/IEEE Frontiers in Education Conference*. Boston, MA.
[9] Cronbach, L. J. 1951. Coefficient alpha and the internal structure of tests. *Psychometrika*. 16: 297–334. doi:10.1007/BF02310555
[10] Dorier, L. 2000. Recherches en Histoire et en Didactique des Mathématiques sur L'Algèbre Linéaire – Perspectives Théoriques sur leurs Interactions. *Les Cahiers du Laboratoire Leibniz*, 12, Grenoble, France.
[11] Fonseca, C., A. Pereira, and J. M. Casas. 2010. The trajectories of study and research as products of didactical engineering: REI en ingeniería didáctica. In *Proceedings of IEEE Information Systems and Technologies* (CISTI), 5th Iberian Conference, Santiago de Compostela, Spain, p. 15.
[12] Gannod, G. C., J. E. Burge, and M. T. Helmick. 2008. Using the inverted classroom to teach software engineering. In *Proceedings of the 30th International Conference on Software Engineering* (ICSE '08), Leipzig, Germany, pp. 777–786. New York, NY: ACM.
[13] González-Martín, A. S., I. Bloch, V. Durand-Guerrier, and M. Maschietto. 2014. Didactic situations and didactical engineering in university mathematics: Cases from the study of calculus and proof. *Research in Mathematics Education*. 16(2): 117–134.
[14] Harel, G. 2000. Three principles of learning and teaching mathematics. In J.-L. Dorier (Ed.), *On the Teaching of Linear Algebra*, pp. 177–189. Grenoble: Kluwer Academic Publishers.
[15] Howard, A. and C. Rorres. 2010. *Elementary Linear Algebra: Applications*, *Tenth Edition*. New York, NY: John Wiley & Sons.
[16] Jolliffe, I. T. 1986. Principal component analysis and factor analysis. In *Principal Component Analysis. Springer Series in Statistics*, pp. 115–128. New York, NY: Springer.
[17] Konyalıoğlu A. C. 2009. An evaluation from students' perspective on visualization approach used in linear algebra instructions. *World Applied Science Journal*. 6(8): 1046–1052.
[18] Lay, D. C. 2005. *Linear Algebra and Its Applications*, *Third Edition*. Boston, MA: Addison-Wesley.
[19] Leon, S. J. 2010. *Linear Algebra with Applications*, *Eighth Edition*. Upper Saddle River, NJ: Prentice-Hall.
[20] Love, B., A. Hodge, N. Grandgenett, and A. Swift. 2014. Student learning and perceptions in a flipped linear algebra course. *International Journal of Mathematical Education in Science and Technology*. 45(3): 317–324. doi:10.1080/0020739X.2013.822582
[21] Lucas, A. 2009. Using peer instruction and I-clickers to enhance student participation in calculus. *PRIMUS*. 19(3): 219–231. doi:10.1080/10511970701643970
[22] Marion, A. L. C. 2007. Métodos de ensino para cursos de administração: uma análise da aplicabilidade e eficiência dos métodos. Master thesis. Programa de Estudos Pós-graduados em Administração de Empresas, Pontifícia universidade Católica, São Paulo, Brazil.

[23] Marsh, H. W. 1982. SEEQ: a reliable, valid, and useful instrument for collecting students' evaluations of university teaching. *British Journal of Educational Psychology*. 52(1): 77–95. doi:10.1111/j.2044-8279.1982.tb02505.x
[24] Mazur, E. 1997. *Peer Instruction a User's Manual*. Upper Saddle River, NJ: Prentice-Hall.
[25] Novak, G., E. Patterson, A. Gavrin, and W. Christian. 1999. *Just-in-Time Teaching: Blending Active Learning and Web Technology*. Upper Saddle River, NJ: Prentice Hall.
[26] Pilzer, S. 2001. Peer instruction in physics and mathematics. *PRIMUS*. 11(2): 185–192. doi:10.1080/10511970108965987
[27] Poole, D. 2005. *Linear Algebra: A Modern Introduction*. 3rd ed. Florence, KY: Cengage Learning.
[28] Porter, L., C. Bailey-Lee, B. Simon, Q. Cutts, and D. Zingaro. 2011. Experience report: A multi-classroom report on the value of peer instruction. In Proceedings of the 16th Annual Joint Conference on Innovation and Technology in Computer Science Education (ITiCSE '11). Darmstadt, Germany, pp.138-142.
[29] Santos, R. 2010. Introdução à Álgebra Linear. July. http://www.mat.ufmg.br/ regi/.
[30] Silva, T. E. V. 2013. Evaluating the teaching effectiveness using multivariate statistics. Master thesis, Department of Teleinformatics Eng., Federal University of Ceará, Fortaleza, Brazil.
[31] Silva, T. E. V., F. H. L. Vasconcelos, A. L. F. Almeida, J. C. M. Mota, and W. B. Andriola. 2013. A new approach to analyze the curriculum structure using the Students' Evaluation of Education Quality instrument. In *Proceedings of IEEE Frontiers in Education Conference* (FIE), Oklahoma City, USA, pp. 171–174.
[32] Strang, G. 2005. *Linear Algebra and Its Applications*, *Fourth Edition*. Boston, MA: Cengage Learning.
[33] Suppapittayaporn, D., N. Emarat, and K. Arayathanitkul. 2010. The effectiveness of peer instruction and structured inquiry on conceptual understanding of force and motion: a case study from Thailand. Research in Science & Technology Education. 28(1): 63–79.
[34] Teixeira K. C. B., T. E. V. Silva, J. C. M. Mota, E. V. O. Teixeira, and N. C. Barroso. 2015. Peer instruction methodology for linear algebra subject: a case study in an engineering course. In Proc. of IEEE Frontiers in Education Conference (FIE), El Paso, pp. 1–7.
[35] Teixeira K. C. B. and J. C. M. Mota. 2016. Active methodologies for teaching linear algebra in an engineering course. IEEE Latin America Transactions, 14, 837–843. doi: 10.1109/TLA.2016.7437230
[36] Teixeira K. C. B., T. E. V. Silva, J. C. M. Mota, E. V. O. Teixeira, and N. C. Barroso. 2018. Effectiveness of Peer Instruction in a linear algebra class within an engineering curriculum. In Press of IEEE Frontiers in Education Conference (FIE), San Jose, pp. 1–7.
[37] Watkins, D. and B. Thomas. 1991. Assessing teaching effectiveness: an Indian perspective. *Assessment & Evaluation in Higher Education*. 16(3): 185–198.
[38] Zimmerman, D. W. 1997. A note on interpretation of the paired-samples *t* test. *Journal of Educational and Behavioral Statistics*. 22: 349–360. doi:10.3102/10769986022003349

BIOGRAPHICAL SKETCH

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