Errata
An Introduction to Multivariable Analysis from Vector to Manifold

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- Exercise 8 on page 81 should be replaced by:

  If $f : \mathbb{R}^M \to \mathbb{R}$ is a differentiable function, then for nonzero vectors $v, w \in \mathbb{R}^M$ and a nonzero scalar $\alpha \in \mathbb{R}$ we have

  $$\|v + w\| D_{v+w} f = \|v\| D_v f + \|w\| D_w f$$

  and

  $$D_{\alpha v} f = \frac{\alpha}{|\alpha|} D_v f.$$

- The condition in Exercise 11 on page 82 is not equivalent to differentiability. Exercises 11 and 12 on page 82 should be replaced by:

  11. Suppose $x \in A^\circ$ and $f : A \to \mathbb{R}^N$ where $A \subseteq \mathbb{R}^M$. If f is differentiable at x, then the following two conditions hold:

  a) For every $h \in \mathbb{R}^M$, $\lim_{\lambda \to 0} \frac{1}{\lambda}(f(x + \lambda h) - f(x))$ exists, where $\lambda \in \mathbb{R}$.

  b) If we set $F(h) = \lim_{\lambda \to 0} \frac{1}{\lambda}(f(x + \lambda h) - f(x))$, then $F : \mathbb{R}^M \to \mathbb{R}^N$ is a linear transformation.

  Show that these two conditions do not imply that f is differentiable at x.

  12. Decide whether or not the following functions $f : \mathbb{R}^2 \to \mathbb{R}$ are differentiable at $(0, 0)$:

  a) $f(u, v) = \begin{cases} \frac{u^2 v^3}{u^4 + v^6} & \text{if } (u, v) \neq (0, 0), \\ 0 & \text{otherwise.} \end{cases}$

  b) $f(u, v) = |uv|$.

- On page 89 (the proof of Theorem 3.3.1) replace the j in

  $$h^{(r)}(\lambda_0) = \left\{[(b - a) \cdot \nabla]^j f\right\}(c)$$

  with an r to obtain

  $$h^{(r)}(\lambda_0) = \{[(b - a) \cdot \nabla]^r f\}(c).$$

- On page 91, in Exercise 4, replace $(0, 0)$ with $(0, 0, 0)$.

- Line four from the top on page 95 should be replaced by:

$$|f(x) - f(y)| = |g(x) + [f'(a)](x) - (g(y) + [f'(a)](y))\,|$$

- The definition of f in Exercise 9 on page 95 should be

$$f(r, \theta) = (r\cos(\theta), r\sin(\theta)).$$

- On page 96, in Example 3.5.1, replace $(f'(r, \theta)) = r$ by $\det(f'(r, \theta)) = r$.
- On page 102, in Example 3.6.2,

$$f(x, y, z) = (0, 0)$$

  should be replaced with

$$f(x, y, z) = (0, 1).$$