Workshop on High-Dimensional Data Fitting and Approximation

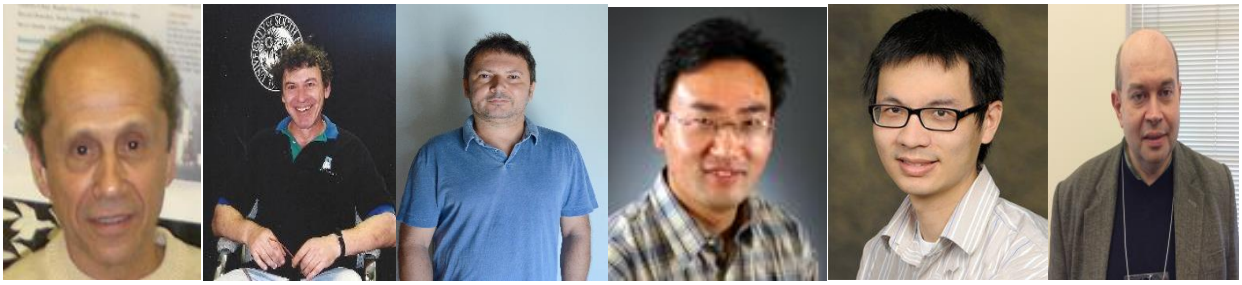
Department of Mathematics, University of Central Florida

March 30, 2017, 9am to 5pm, MSB 318

Data gathering, processing and management have become omnipresent in our society. Most of data gathered have very high dimensions and complex geometric structure. It calls for novel mathematical techniques to deal with explosive growth of dimensionality.

Computational linear algebra has successfully applied to solve massive data processing problems, computational harmonic analysis has developed dictionaries for adaptive sparse representation, and high-dimensional approximation theory could crack the curse of dimensionality.

The one-day workshop will concentrate on data fitting and approximation. During the workshop, Professor Charles A. Micchelli (Distinguished professor at New York University at Albany and an invited speaker of International Congresses of Mathematician, Warsaw 1983) will talk about high-dimensional data fitting using discrete least squares. Professor Boris Shekhtman from University of South Florida will provide you a taste of ideal interpolation concerning projection. Professor Plamen Simeonov from University of Houston Downtown will bring a polynomial blossom for the Askey-Wilson operator. Professor Yi Wang from Auburn University at Montgomery will discuss sparse representation of signals with non-linear Fourier atoms. Professors Teng Zhang will consider well-tempered landscape for non-convex robust subspace recovery, and Professor Alexander Tovstolis will present some sharp results on approximating smooth functions by entire functions of exponential type.



The workshop is organized by Mourad Ismail, Charles Micchelli and Qiyu Sun and it is partially supported by the Department of Mathematics and the National Science Foundation.

One Day Workshop on

High Dimensional Data Fitting and Approximation Theory

Department of Mathematics, University of Central Florida

March 30, 2017, Math Science Building 318

09:00—09:05 Opening

09:05—09:55 A taste of ideal interpolation
Boris Shekhtman (University of South Florida)

10:00—10:50 On multivariate discrete least squares
Charles Micchelli (University at Albany)

11:00—12:20 Lunch Break

12:30---01:20 A well-tempered landscape for non-convex robust subspace recovery
Teng Zhang (University of Central Florida)

01:30—02:20 Sparse representation of signals with nonlinear Fourier atoms
Yi Wang (Auburn University at Montgomery)

02:30—02:50 Tea Break

02:50—03:40 A polynomial blossom for the Askey-Wilson operator
Plamen Simeonov (University of Houston, Downtown)

03:50—04:40 Approximation of Smooth Functions on Half-Axis and the complement of (-1,1)
Oleksandr Tovstolis (University of Central Florida)

Organizer: Mourad Ismail, Charles Micchelli and Qiyu Sun

Titles and Abstracts for
Workshop on high dimensional data fitting and approximation
March 30, 2017, University of Central Florida

Boris Shekhtman (shekhtma@usf.edu) – University of South Florida

A taste of ideal interpolation

Abstract: An ideal interpolation is an elegant generalization of Hermite interpolation to the multivariate setting proposed by Garrett Birkhoff and further popularized by Carl de Boor.

Ideal interpolation concerns projectors P on the space of polynomials in several variables such that the kernel of P is an ideal. In one variable, these are precisely the Hermite interpolation projectors. Once such a generalization is made, it is a natural yet completely futile desire to extend the properties of Hermite projectors to their multivariate analogs. In this talk, I will outline the properties of ideal projectors and their relation to algebraic geometry. I will also present several results (mostly negative) and related open problems about ideal projectors.

Charles A. Micchelli (charles_micchelli@hotmail.com) – University at Albany

On multivariate discrete least squares

Abstract: Given m smooth functions in n dimensions we consider the discrete least squares problem of finding a linear combination of these functions which best fits scalar data at prescribed points in n dimensions. We investigate two problems. What happens when the multivariate points are all near the origin and as well what happens when a new value of the prescribed linear combination is also close to the origin.

Teng Zhang (teng.zhang@ucf.edu) – University of Central Florida

A well-tempered landscape for non-convex robust subspace recovery

Abstract: We present a mathematical analysis of a gradient descent method for Robust Subspace Recovery. The optimization is cast as a minimization over the Grassmannian manifold, and gradient steps are taken along geodesics. We show that under a generic condition, the energy landscape is nice enough for the non-convex gradient method to exactly recover an underlying subspace. The condition is shown to hold with high probability for a certain model of data.

Yi Wang (ywang2@aum.edu) – Auburn University at Montgomery

Sparse representation of signals with non-linear Fourier atoms

Abstract: In this paper, we study the sparse representation of a finite energy signal with intrinsic mode functions in a dictionary consisting of non-linear Fourier atoms. Each non-linear Fourier atom is a mono-component with a physically meaningful non-negative instantaneous frequency. The sparse representation is

obtained adaptively by an orthogonal matching pursuit using a two-level greedy search. It is demonstrated that the representation has efficient energy decay in error compared to the Fourier expansion and wavelet expansion.

Plamen Simeonov (simeonovp@uhd.edu) – University of Houston, Downtown

A polynomial blossom for the Askey-Wilson operator

Abstract: We introduce a polynomial blossom which in addition of having the standard symmetry and multi-affine properties, has a diagonal property specifically designed for evaluating Askey-Wilson derivatives of polynomials. We describe standard recursive evaluation algorithms based on this type of blossom. We also introduce a corresponding Bernstein polynomial basis, and establish a dual functional property which is used to derive several identities and properties for this basis, including a Marsden identity.

Alexander (Oleksandr) V. Tovstolis (joint work with Xin Li) (Oleksandr.Tovstolis@ucf.edu) – University of Central Florida

Approximation of smooth functions on $[1,\infty)$ *and* $\mathbb{R}\setminus(-1,1)$ *by entire functions of exponential type*

Abstract: In 1946, Sergey M. Nikolskĭi discovered an effect of better pointwise approximation of a smooth function by algebraic polynomials. Namely, for a function from Sobolev class $W^1[-1,1]$, there is a sequence of algebraic polynomials $\{p_n\}_{n=0}^{\infty}$, $p_n \in \mathbb{R}_n[x]$, such that

$$|f(x)-p_n(x)| \le \frac{\pi}{2}\frac{\sqrt{1-x^2}}{n+1} + O\left(\frac{\ln(n+2)}{(n+1)^2}\right).$$

The constant $\pi/2$ in the first term cannot be improved.

There are several generalizations of this result. The most recent one is due to Roald M. Trigub (1993), where the asymptotically sharp estimate was obtained for the $W^r[-1,1]$ class.

We will focus on pointwise approximation of a function from the Sobolev class $W^{r,\infty}(\mathbb{R}\setminus(-1,1))$ by entire functions of exponential type at most σ. Known estimates of such approximation is due to Ju. A. Brudnyĭ (1959). However, the result is not valid as stated. We found a fix. As in the Trigub's article, we deduce our estimates from the corresponding sharp result on the uniform approximation. In our case, this is the Akhiezer's theorem on uniform approximation (on $\mathbb{R}$) of a function from $W^r(\mathbb{R})$ class by entire functions of exponential type σ. Some useful trick from the Ju. A. Brudnyĭ's proof also plays an important role.