The Seventh Hangzhou Workshop on Harmonic Analysis and Applications

December 16–18, 2022

Zhejiang University of Science and Technology

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Titles and Abstracts

Local potential operator and uniform resolvent estimate for generalized Schrodinger operator in Orlicz spaces

Jun Cao
Zhejiang University of Technology

In this talk, we consider the local potential operator with integral kernel restricted in a ball of radius less than some fixed number $r \in (0, \infty)$ has appeared frequently in the spectral estimates of Schrödinger operator. We establish a good-λ inequality for this operator and characterize its uniform boundedness on the weighted Orlicz space in both strong and weak senses. The uniformity in r of this boundedness enable us to recover the classical boundedness of "global" potential operator, by letting $r \to \infty$. As an application, we establish uniform estimate for the resolvent $(\lambda - \mathcal{L})^{-1}$ of some generalized Schrödinger operator $\mathcal{L} := \mathcal{L}_0 + V$ on Orlicz space. An explicit representation in its operator norm on the dependence of $\lambda \in (0, \infty)$ is also given.

The Garnett-Jones Theorem on BMO space associated with operators

Peng Chen
Sun Yat-sen University

Let BMO_L be the BMO space associated with an operator L. In this talk, we show two kinds of decomposition of BMO_L functions. As an application, we prove the Garnett-Jones Theorem on BMO_L, that is, comparable upper and lower bounds are given for the distance for a BMO_L function from L^∞ space. The other decomposition claims that a compact supported BMO_L function can be decomposed as the summation of an L^∞ function and the integral of the heat kernel with respect to a finite Carleson measure. Random dyadic lattice method is used in both of the two kinds of decomposition. All condition we need is the heat kernel of the semigroup generated by L satisfies the Gaussian upper bound.

Stable image reconstruction by TV type methods

Wengu Chen
Institute of Applied Physics and Computational Mathematics

Transformed L1 (TL1) regularization has been shown to have comparable signal recovery capability. In the spirit of the TL1 method, we introduce a transformed total variation (TTV) minimization model to investigate robust image recovery from a certain number of noisy measurements by the proposed TTV minimization model. An optimal error bound, up to a logarithmic factor, of robust image recovery from compressed measurements via the TTV minimization model is established, and the RIP based condition is improved compared with total variation (TV) minimization. Numerical results of image reconstruction demonstrate our theoretical results and illustrate the efficiency of the TTV minimization model among state-of-the-art methods.

Some properties of generalized singular integrals related to SQG equation

Yanping Chen
University of Science and Technology Beijing

We will give some properties of generalized singular integrals to SQG equations.

Recent progress on Schatten classes and Riesz transform commutators

Zhijie Fan
Wuhan University

This talk is about recent progress on Schatten classes and non-Euclidean Riesz transform commutators. By developing a new approach to bypass the use of Fourier analysis and the standard dyadic structure of Euclidean space, we show that the Schatten norm of several kinds of non-Euclidean Riesz transform commutators can be characterized in terms of Besov norms of the symbol.

Norm-controlled inversion for algebra of operators

Qiquan Fang
Zhejiang University of Science and Technology

Matrices and integral operators with off-diagonal decay appear in numerous areas of mathematics including numerical analysis and harmonic analysis, and they also play important roles in engineering science including signal processing and communication engineering. Wiener's lemma states that the localization of matrices and integral operators are preserved under inversion. In this talk, we consider the stability and norm-controlled inversion for matrix and integral operators.

Singular integrals and wavelet-type decomposition with applications to Littlewood-Paley theory and Hardy space in Dunkl setting

Yongsheng Han
Auburn University, USA

We establish the wavelet-type decomposition in the Dunkl setting which is associated with finite reflection groups on the Euclidean space. The main tool is to introduce a new class of singular inegral operators and provide the $T1$ theorem, This group structure induces two nonequivalent metrics: the Euclidean metric and the Dunkl metric, which are both involved in the estimates of singular integrals, heat and Poisson kernels in this Dunkl setting. As an application, we establish the Littlewood–Paley theory and Hardy space in the Dunkl setting. The L^2 boundedness plays a crucial role. New tools developed include the weak-type discrete Calderón reproducing formulae, Coifman-type approximation to the identity, Meyer-type commutation Lemma, new almost orthogonal estimates in the Dunkl setting and the wavelet-type decomposition, and the molecular decomposition of the Dunkl–Hardy space.

Yamabe type equations on lattice graphs

Bobo Hua
Fudan University

Yamabe type semilinear PDEs have been well studied on $\mathbb{R}^n$. We discuss known results and open problems on the lattice graph $\mathbb{Z}^n$. This is based on joint work with Ruowei Li and Florentin Muench, Hichem Hajaiej and Fengwen Han.

Sharp uncertainty principles and their stability

Nguyen Lam
Memorial University of Newfoundland, Canada

The Heisenberg uncertainty principle, which is a fundamental result in quantum mechanics, and related inequalities such as the hydrogen and Hardy uncertainty principles, belong to the family of geometric inequalities known as the Caffarelli-Kohn-Nirenberg inequalities. In this talk, we discuss some recent results about the optimal uncertainty principles, Caffarelli-Kohn-Nirenberg inequalities, and their quantitative stability.

Maximal function associated to space curves

Sanghyuk Lee
Seoul National University, Republic of Korea

This talk concerns L^p bound on the maximal function generated by averages over space curves. A most typical example is Bourgain's circular maximal theorem in 2 dimensions. However, such maximal bounds have not been known for the curves in higher dimensions until recently. We discuss recent developments in the problem.

Dantzig selector: signal recovery from phaseless measurements

Peng Li
Lanzhou University

Consider the task of recovering a complex vector $\mathbf{x}_0$ from phaseless nonlinear measurements $\mathbf{b}$ contaminated by a random noise $\mathbf{e}$. This nonconvex problem is called complex phase retrieval. Especially, when the unknown vector $\mathbf{x}_0$ is sparse, it is called complex sparse phase retrieval.营n this talk, we first consider the ∞-norm data fitting item model– the nonlinear Dantzig selector. We show that the stable recovery error estimation can be obtained with high probability. We also show the difference between our model and the 1-norm data fitting item model-Nonlinear RLAD, and the 2-norm data fitting item model—Nonlinear Lasso.

L^p-improving bounds and weighted estimates for maximal functions associated with curvature

Wenjuan Li
Northwestern Polytechnical University

In this paper, we establish weighted estimates for a wide class of maximal functions along some finite type curves and hypersurfaces. In particular, various impacts of non-isotropic dilations are considered. Via the methodology of sparse domination, the weighted estimates for the global maximal functions can be reduced to the L^p-improving properties of the corresponding localized maximal functions. We mainly focus on proving the $L^p \to L^q$ bounds ($q > p$) for localized maximal functions with non-isotropic dilations of curves and hypersurfaces whose curvatures vanish to finite order at some points. As a corollary, we obtain the weighted inequalities for the corresponding global maximal functions, which generalize the known unweighted estimates.

Sampling and reconstruction in reproducing kernel spaces with mixed norm

Yaxu Li
Hangzhou Normal University

In this talk we consider (non)random sampling of signals in (in)finite-dimensional reproducing kernel spaces with mixed norm. We first assume that the signals of interest reside in a reproducing kernel space with mixed norm defined on metric measure spaces. We consider three

sampling schemes of the signals of interest depending on the accessible sampling domains that are the whole metric measure spaces or part thereof. We propose iterative algorithms that reconstruct or approximate the original signals from the sampling data. When one can do sampling on the whole metric measure spaces, the proposed algorithm can perfectly reconstruct the original signals if the sampling size is large enough. When one cannot access the whole metric measure spaces, we show that the proposed algorithms can approximate the original concentrated signals with the approximation error being a multiple of the concentration ratio if the sampling set has sufficient density. Then we consider random sampling of signals in (in)finite-dimensional reproducing kernel spaces with mixed norm. Here the random sampling refers to randomly taken sampling positions according to some probability measure. We study the stability of random sampling procedure by establishing sampling inequality that holds with high probability when the sampling size is large. We establish the probabilistic sampling inequality though a combination of mathematical analysis and probabilistic analysis. The main tools we use are covering number of signal(function) space and (uniform) large deviation inequality for a sequence of random variables. We provide a concise proof and our proof leads to explicit and transparent estimates involved in the probability with which the sampling inequality holds.

Area integrals and maximal functions associated with the Dunkl operators

Zhongkai Li
Shanghai Normal University

In this talk I shall present some recent researches about Lusin-type area integrals associated to the Dunkl operators and the non-tangential maximal function on the upper half-space.

Regularity properties of maximal commutator and its fractional variant

Feng Liu
Shandong University of Science and Technology

In this talk, I'll present some recent results concerning regularity properties of maximal commutator and its fractional variant, both in global case and in local case.

Boundedness of fractional heat semigroups generated by degenerate Schrödinger operator

Yu Liu
University of Science and Technology Beijing

In this talk, we introduce various regularity estimates about the fractional heat semigroup $\{e^{-tL^\alpha}\}_{t>0}$, where L^α denote the fractional powers of L for $\alpha \in (0,1)$ and L is a a degenerate

Schrödinger operator in $\mathbb{R}^n$. Moreover, we obtain the boundedness on the weighted Morrey spaces and BMO type spaces for some operators related to L.

Stability of some geometric and functional inequalities

Guozhen Lu
University of Connecticut, USA

In this talk, we will discuss recent works on stability of some geometric and functional inequalities, including the Heisenebrg uncertainty principle, Hydrogen uncertainty principle, and L^2 and L^p Caffarelli-Kohn-Nirenberg inequalities in Euclidean spaces (joint works with A. Do, C. Cazacu, J. Flynn, N. Lam). If time allows, we will also discuss the stability of the Moser-Onofri and log Sobolev inequalities on the sphere (joint works with L. Chen and H. Tang).

Elliptic boundary problems for edge-degenerate pseudodifferential operators

Zhuoping Ruan
Nanjing University

Let X be a compact C^∞ manifold with boundary, Y. A distribution u on $X \setminus Y$ is said to have a conormal asymptotic expansion of type P if, formally,

$$u(x,y) \sim \sum_{(p,k)\in P} \frac{(-1)^k}{k!}\, x^{-p} \log^k x\, u_{pk}(y) \quad \text{as } x \to +0,$$

where x is a boundary defining function and $u_{pk} \in C^{-\infty}(Y)$. Given asymptotic types P, Q, we construct a calculus for edge-degenerate pseudodifferential operators A on $X \setminus Y$ with the property that if u has an asymptotic expansion of type P, then Au has an asymptotic expansion of type Q. The calculus contains the trace operators γ_{pk}, where $\gamma_{pk}u = u_{pk}$, while elliptic elements possess parametrices within the calculus. For $P = Q$ being the asymptotic type resulting from a Taylor series expansion at $x = 0$, we recover Boutet de Monvel's calculus. This is joint work with Xiaochun Liu (Wuhan) and Ingo Witt (Göttingen).

一个奇异积分交换子的端点估计

Songbai Wang
Chongqing Three Gorges University

我们获得了一个奇异积分交换子的新的端点估计，拓展了[J. Funct. Anal., 128: 163-185 (1995)]的一个结果。同时，应用外插定理，我们也得到了奇异积分交换子在Orlicz空间上的一个端点估计。

Wavelet characterizations of operator-valued Hardy spaces

Xinfeng Wu
China University of Mining and Technology

Let $\mathcal{M}$ be a von Neumann algebra equipped with a normal semifinite faithful trace τ , and $H_p(\mathbb{R}^n, \mathcal{M})$ be the operator-valued Hardy spaces introduced by Tao Mei. We characterize the operator-valued column Hardy space $H_p^c(\mathbb{R}^n, \mathcal{M})(1 \leq p < \infty)$ by using several square functions involving wavelets, which corresponds to Meyer's wavelet characterizations of the classical Hardy space when $p = 1$. This is joint work with Guixiang Hong and Wenhua Wang.

The minimizers of the p-frame potential

Zili Xu
The Hong Kong University of Science and Technology

Minimal potential energy problems have been actively discussed over the last decades in connection with applications in physics, signal analysis and numerical integration. Generally, one aims to find distributions of N points on the unit sphere which minimize the potential energy over all sized N configurations. One of the most interesting potential energies is the p-frame potential. For any positive real number p, the p-frame potential of N unit vectors $X := \{\mathbf{x}_1, \ldots, \mathbf{x}_N\} \subset \mathbb{R}^d$ is defined as $\mathrm{FP}_{p,N,d}(X) = \sum_{i \neq j} |\langle \mathbf{x}_i, \mathbf{x}_j \rangle|^p$. In this talk, we focus on this quantity for $N = d+1$ points and establish uniqueness of minimizers of $\mathrm{FP}_{p,d+1,d}$ for all $p \in (0, 2)$. Our results completely solve the minimization problem of the p-frame potential when $N = d + 1$, confirming a conjecture posed by Chen, Gonzales, Goodman, Kang and Okoudjou.

On the boundedness of multilinear pseudo-differential operators

Qingying Xue
Beijing Normal University

The origin of the study of the linear pseudo-differential operators can be traced back to the outstanding works of Kohn and Nirenberg and Hörmander. The theroy of multilinear pseudo-differential operators was first established by Coifman and Meyer. Since then, many works have been done. In this talk, we first recall some background of the multilinear pseudo-differential operators and then present some recent results, in particular, the quantitative weighted estimates for the multilinear pseudo-differential operators in general function spaces.

Triple non-linear structure and Marcinkiewicz theorem

Qixiang Yang
Wuhan University

Recently we've been focusing on two 50-year-old conjectures. We've solved both of these conjectures completely, and here's the second one. We reveal the triple non-linear structure of the K functional of real interpolation spaces for general Besov type spaces by applying vertex K functional defined by wavelets. The techniques used in this report simply open a window to interpolation in other more complex spaces. Real interpolation can substantially improve operator continuity up to some log order in many cases. So it has been getting attention. In 1950, Lorentz introduced Lorentz spaces by using rearrangement functions. In 1964, Hunt extend Marcinkiewicz theorem to Lorentz spaces. In this report, we systematically extend their work to generalized Besov spaces composed of hierarchical quasi-Banach spaces, solving the second conjecture about real interpolation proposed by Peetre in his book in the 1970s.

The endpoint theory of fractional ball Banach Sobolev spaces

Yangyang Zhang
Beijing Normal University

This report mainly introduces the endpoint theory of fractional ball Banach Sobolev spaces. Let $p \in [1,\infty)$ and X be a ball Banach function space on $\mathbb{R}^n$ with an absolutely continuous norm for which the Hardy囧ittlewood maximal operator is bounded on $(X^{1/p})\prime$, the associate (dual) space of its $1/p$-convexification. In this case, the fundamental formula holds true

$$\lim_{s\to 1-}(1-s)\left\|\left[\int_{\mathbb{R}^n}\frac{|f(\cdot)-f(y)|^p}{|\cdot-y|^{n+sp}}dy\right]^{1/p}\right\|_X^p=\frac{2\pi^{(n-1)/2}\Gamma((p+1)/2)}{p\Gamma((p+n)/2)}\||\nabla f|\|_X^p$$

for any $f\in X$, where $\Gamma?$ is the Gamma function. This identity coincides with the celebrated classical formula of Bourgain, Brezis, and Mironescu. Moreover, for any locally integrable function f with $\||\nabla f|\|_x<\infty$,

$$\sup_{0<\lambda<\infty}\left\|\left|\left\{y\in\mathbb{R}^n:|f(\cdot)-f(y)|>\lambda|\cdot-y|^{n/q+1}\right\}\right|^{1/q}\right\|_X\sim\||\nabla f|\|_X$$

with the positive equivalence constants independent of f, where the index $q\in(0,\infty)$ is related to X. In particular, when $X:=L^p(\mathbb{R}^n)$ with $p\in[1,\infty)$, the above formulae hold true for any given $q\in(0,\infty)$ with $n(1/p-1/q)<1$, which when $q=p$ are exactly the recent surprising formulae of H. Brezis, J. Van Schaftingen, and P.-L. Yung.

Weighted estimates for product singular integral operators in Journé's class on RD-spaces

Taotao Zheng
Zhejiang University of Science and Technology

This talk will give the Plancherel-Pôlya characterization of product weighted Triebel-Lizorkin spaces and product weighted Besov spaces on RD-spaces and make some estimates for the

product singular integral operators in Journé's class on these function spaces. As a result of this, we present some sufficient conditions for the boundedness of product singular integral operators on the product Lipschitz spaces and weighted Hardy spaces.

The trilinear Stein-Weiss inequality with its optimal constants

Yongliang Zhou
Harbin Engineering University

For $f \in mathcalS(\mathbb{R}^n)$, the solution to the equation $(-\triangle)^{(n-\lambda)/2}(|x|^\beta u(x)) = f(x)|x|^{-\alpha}$ can be written formally as

$$u(x) = S_{\alpha,\lambda,\beta}(f)(x) = \int_{\mathbb{R}^n} \frac{f(y)}{|y|^\alpha |x-y|^\lambda |x|^\beta} dy$$

For $f \in L^p(\mathbb{R}^n)$, the boundedness of the operator$S_{\alpha,\lambda,\beta}$ from $L^p(\mathbb{R}^n)$ to $L^q(\mathbb{R}^n)$ with $p \leq q$ is the classical Stein-Weiss inequality, which is also known as the doubly weighted Hardy-Littlewood-Sobolev inequality. They can also be viewed as bilinear inequalities. This report will consider the general trilinear Stein-Weiss inequality, including the sufficient and necessary conditions for the inequality to hold and the best constants in the inequality for some special cases.
