CURRICULUM LEARNING FOR INVERSE SCATTERING

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Abstract

- We consider the inverse acoustic obstacle scattering problem for sound soft obstacles in two dimensions where the boundary of the obstacle is determined from measurements of the scattered field at a set of receivers placed around the obstacle.
- In [1], the authors presented a novel neural network (NN) scheme that performed better than the classical LSM in the search of the initial guess.
- In this work, we present a new strategy based in curriculum learning to train the neural network developed in [1].

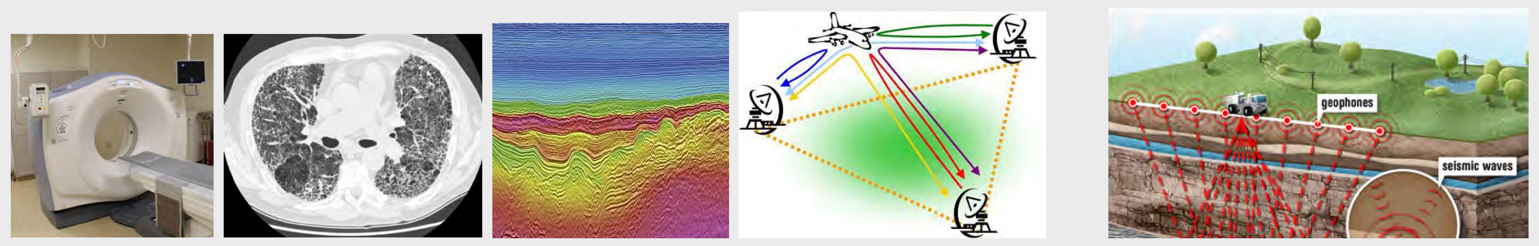

Fig. 1: Real World Application: Medical and Seismic Imaging, Multistatic System

Background

The scattering process can be modeled by the PDE:

$$\begin{cases} \Delta u + k^2 u = 0 & \text{on } \mathbb{R} \setminus \bar{\Omega} \\ u = 0 & \text{on } \partial\Omega \\ \lim_{|x|\to\infty} |x|^{1/2} \left(\frac{\partial}{\partial |x|} - ik\right) u^{scat}(x) = 0 & \end{cases}$$

where we have that $u = u^{inc} + u^{scat}$, where u^{inc} is the incident wave and u^{scat} is the scattered wave.

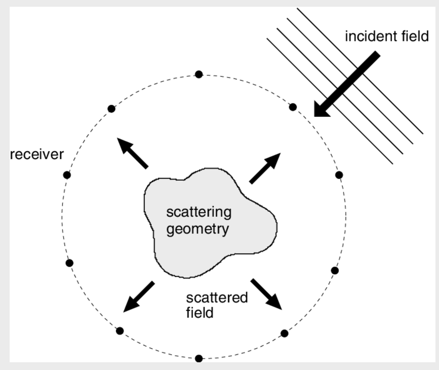

Fig. 2: Setting of the Problem

Forward Problem:
Given u^{inc} and an object with boundary Ω, we solve for the scattered fields represented as a linear combination of single and double layer potentials. We can use the Theory of Integral Equations and Potential Theory to solve the equation and get:

$$u^{scat}(x) = D\varphi(x) + i\eta S\varphi(x), \text{ on } x \in \mathbb{R}^2 \setminus \bar{\Omega} \text{ and } [(\tfrac{1}{2}I + D + i\eta S)\varphi](x) = -u^{inc}(x)$$

Inverse Problem:
Given u^{inc} and u^{scat}, we are looking for $\hat{\Gamma} = \partial\Omega$ such that: $\hat{\Gamma} = \underset{\Gamma}{\operatorname{argmin}}\{\|F(\Gamma) - u^{scat}\|\}$.

Methods like recursive linearization algorithm (RLA) used to solve Inverse Scattering Problems work very well but require an initial guess to function. The more complex the boundary, the better the initial guess must be in order to converge on a usable solution.

LSM vs. Neural Network

LSM:
The linear sampling method (LSM) [1] is a sampling method where the level-set of an indicator function is used for the solution of the inverse obstacle scattering problem. We define the far field operator $\mathcal{L}$ as

$$\mathcal{L}[g](x) = \int_0^{2\pi} u_{scat}(x,\theta) g(\theta) d\theta \,,$$

and the L is the discretized counterpart of this operator. In the LSM, for each point x in the domain we solve the problem

$$(L^*L + \alpha I) g_x = f_x,$$

α is a regularization parameter, $(f_x)_i = G(k|x - x_i|$, G is the Green's function, and x_i is the $i-$th sensor. A point is in the domain if g_x is above a threshold.

Neural Network to substitute the LSM:

- In [4], the authors proposed a NN that would simulate the results of the LSM. The main idea would be to train this network using the same data available for the LSM.
- The NN presented results superior to the LSM in most examples.

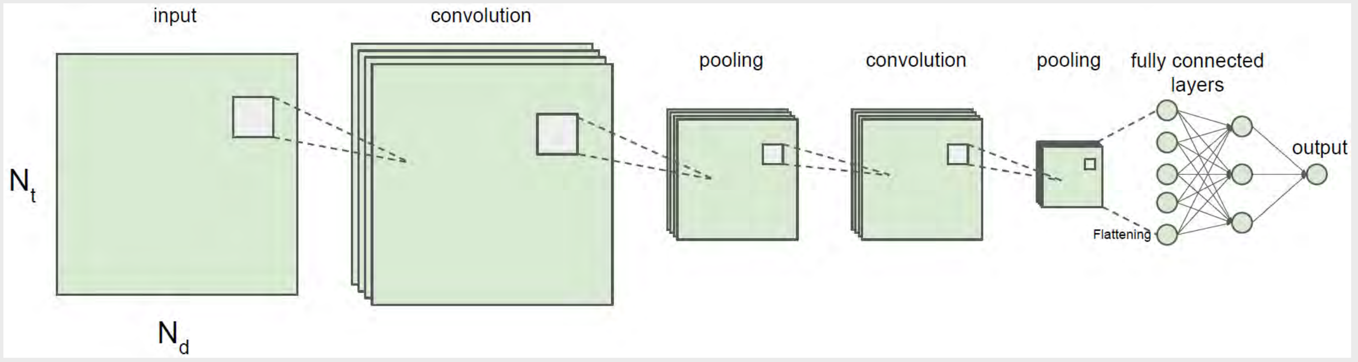

Fig. 3: Neural Network structure that beat LSM

Curriculum Learning

- Curriculum learning (CL) trains the model by transitioning gradually from "easy" to "hard" data, rather than randomly selected data like traditional machine learning (ML) schemes. In our research, "easy" data refers to lower frequency waves, while "hard" data refers to higher frequency waves.
- We will use CL to provide the initial guess for the iterative solvers and compare the results to LSM warm starts.

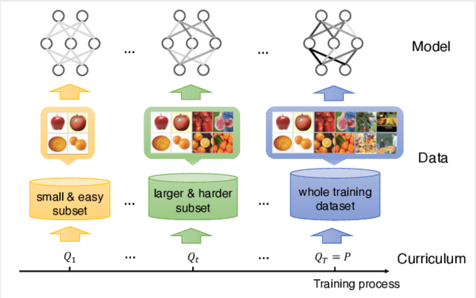

Fig. 4: Visual of the Problem

Graphical Results

Experiment 1: Reconstructions

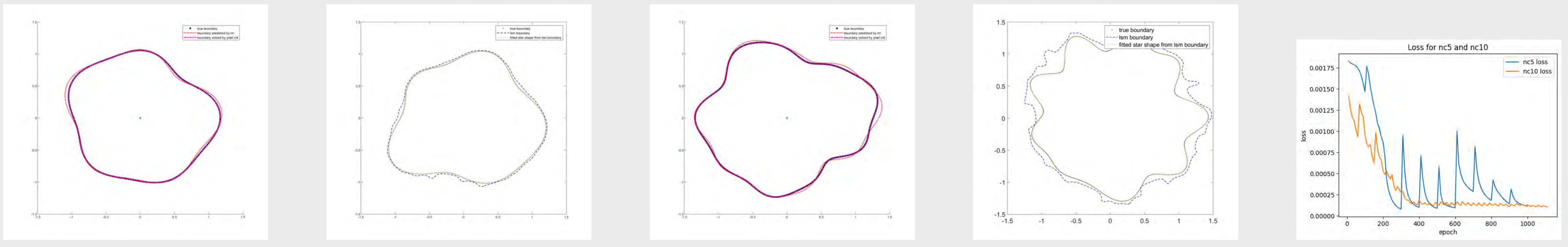

Fig. 2: The CL initial guess and its inversion compared to LSM's for nc5 and nc10 as well as the training loss for the networks

Experiment 2: Robustness to Noise

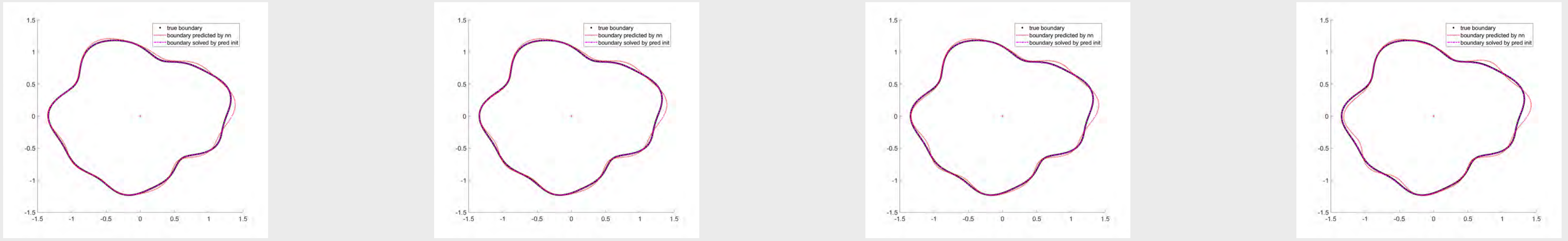

Fig. 3: Varying Noise Levels for nc10 (0%, 10%, 25%, 50%)

Experiment 3: Robustness to General Domains

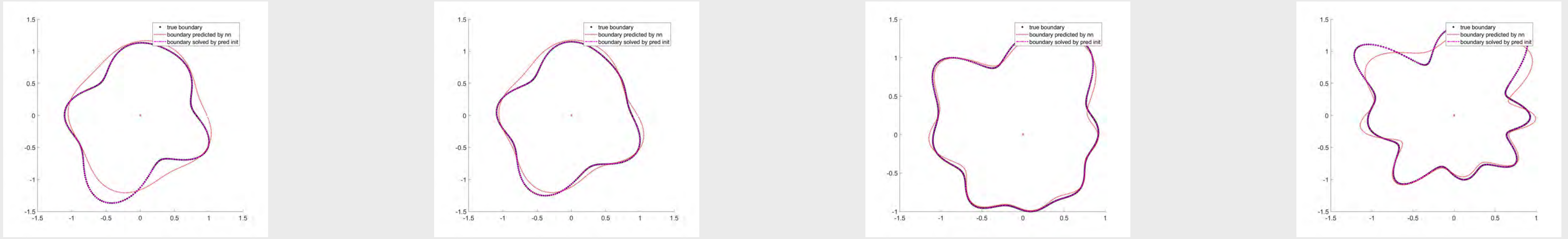

Fig. 4: Reconstructions with domains outside of training interval

Conclusions

Our results show that the use of the curriculum learning strategy to train the neural network provided an improvement in the choice of a warm-start to use in the reconstruction of the obstacle shape in comparison with traditional methods such as LSM.

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