Theory and Applications of Pricing Financial Derivatives

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Abstract

This research centers around the valuation of financial derivatives by investigating various approaches including the construction of Binomial and Trinomial Models, solving the Black-Scholes Model, and so on. Then we intend to gather data from real markets, such as stocks, options, currencies, and bonds markets, and then perform some analysis to analyze the data. The studies on this topic build the connection between financial mathematics and applications and are hence applicable to the area of financial practice.

Background

Definitions:

- ***Option*** represents the right to buy or sell an asset for a fixed price for a limited amount of time.
- ***Call*** an option to buy an asset for a fixed price on or before the expiration date.
- ***Put*** an option to sell an asset for a fixed price on or before the expiration date.

Discrete Models

To begin the valuation of an option, we start with basic discrete models.

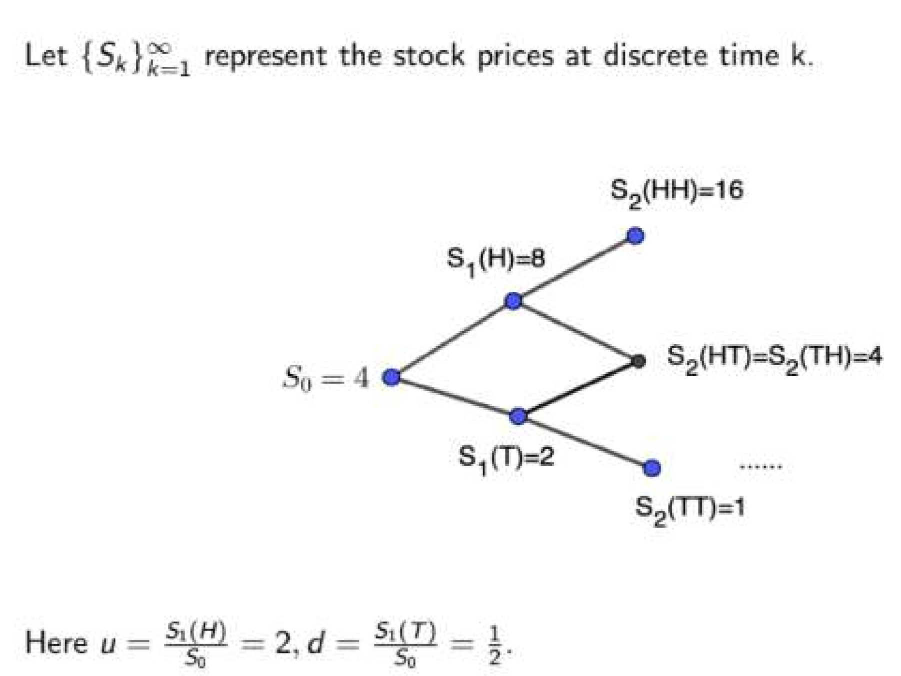

Figure 1. Binomial Tree

- **Binomial Tree** A binomial tree is the most simple representation of the intrinsic values an option may take at different time periods. This model assumes two possible outcomes. Either an upward movement U or a downward movement D.
- **Trinomial Tree** A trinomial tree differs from a binomial tree by offering another possible outcome. Instead of only taking into account whether the value goes up or down the trinomial tree includes the possibility the value remains unchanging.

Heat Equation

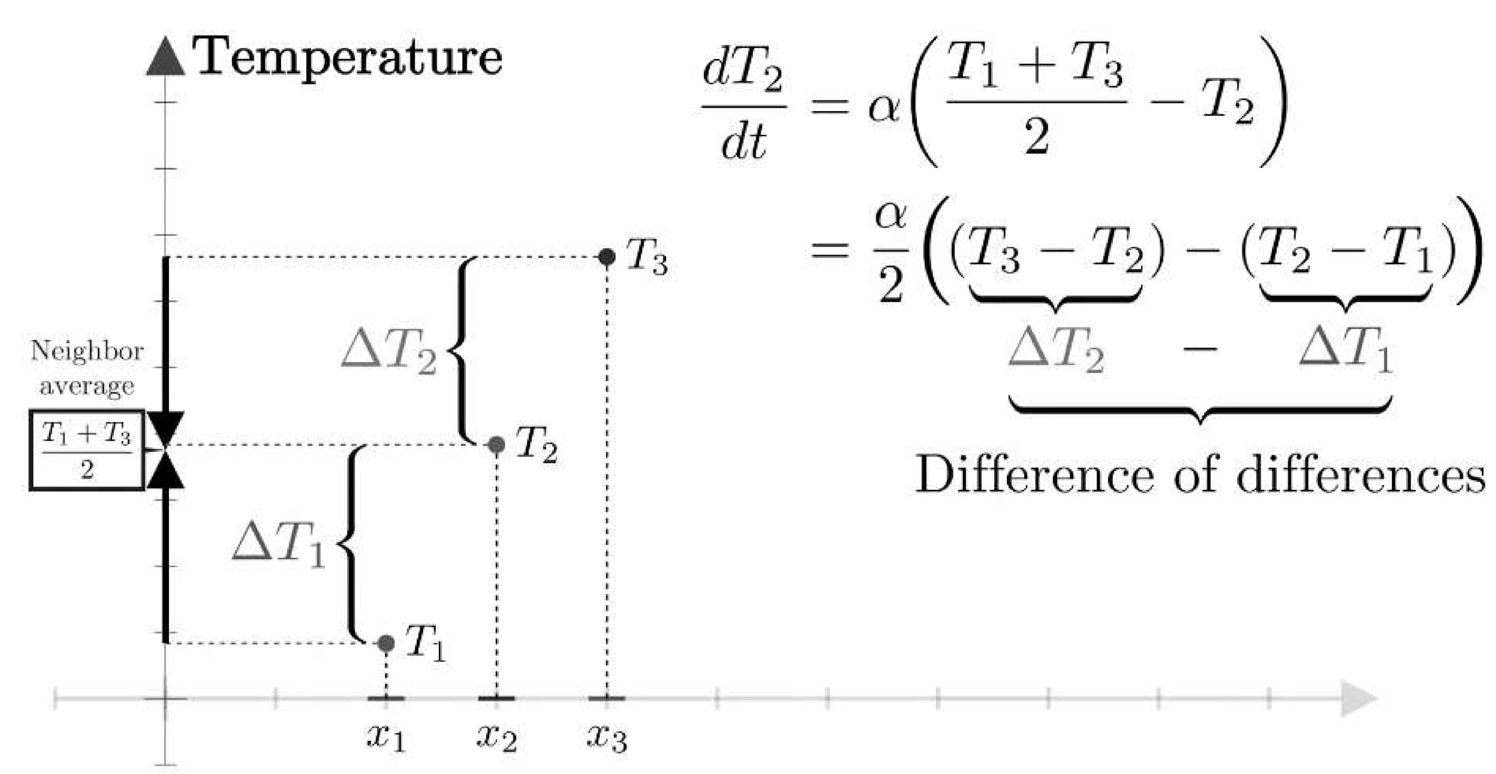

Figure 2. Difference of Differences

The one dimensional heat equation is:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

and the solution, obtained through the finite difference method, represents the temperature at different points at different times. An example solution, based on a specific boundary and initial condition, is as follows:

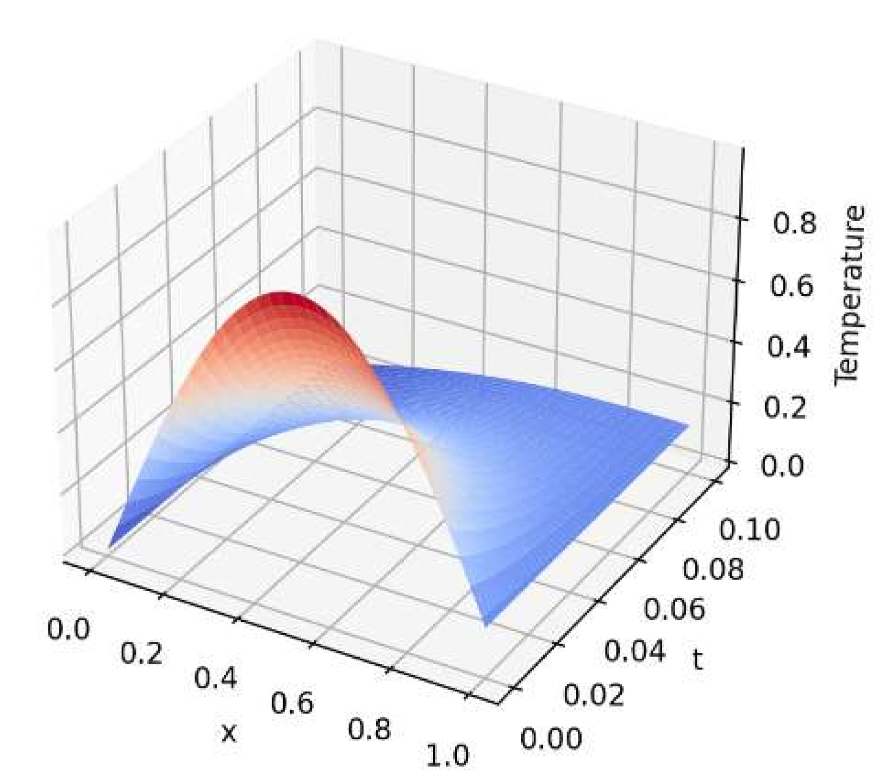

Figure 3. Heat Equation Solution

Black–Scholes Model

The Black-Scholes starts with this equation:

$$\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + rS\frac{\partial C}{\partial S} - rC = 0$$

where C is the price of the option as a function of the stock price S and time t, r is the risk-free interest rate, and σ is the volatility of the stock.

Current Results

Solving the corresponding heat equation for the Black-Scholes model and using theoretical data, we get:

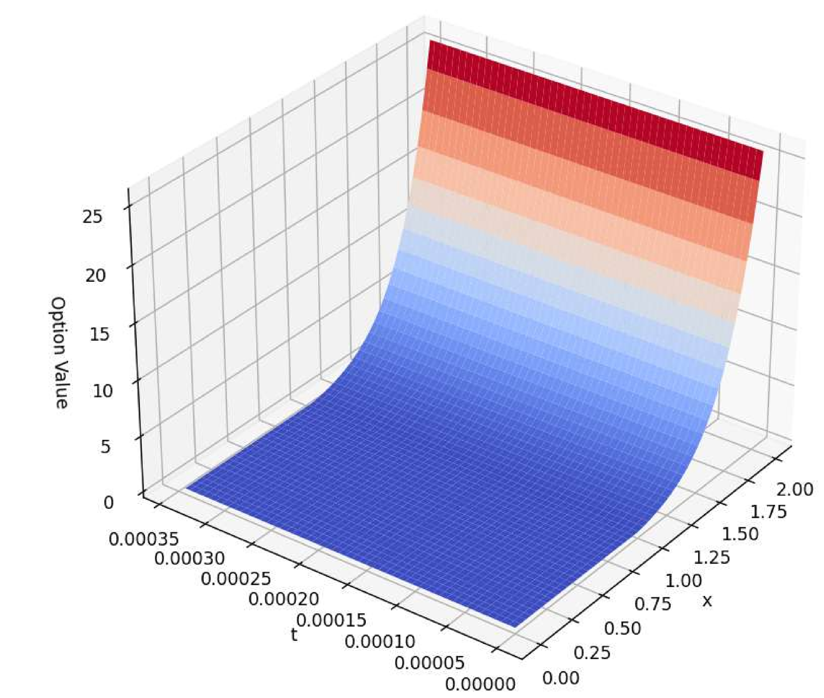

Figure 4. Option Values

Testing real-world data showed as follows:

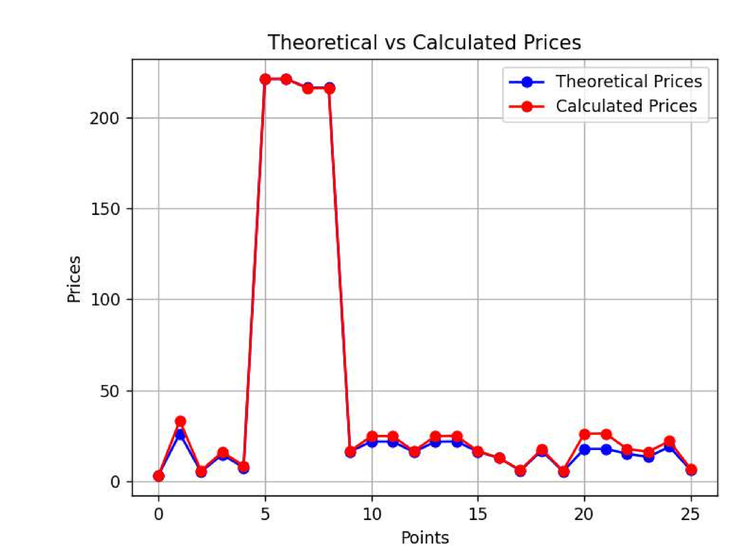

Figure 5. Theoretical vs. Calculated Prices

Future Research

Some research that we were not able to get but would be of some interest includes currencies and sentimental analysis. Looking into other financial derivatives such as currencies is something that should be looked at. Furthermore, sentimental analysis of how public opinions can cause variations from predicted projections would also be supplemental to this research.

References

Dr. Li. 2023. "REU Week 5." Finance Mathematics, University of Central Florida.

Dr. Li. 2023. "Finance Lecture." Page 35. University of Central Florida.

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Figure 2, Grant Sanderson