Accelerating the Iterative Solution of the Forward Medium Scattering Problem

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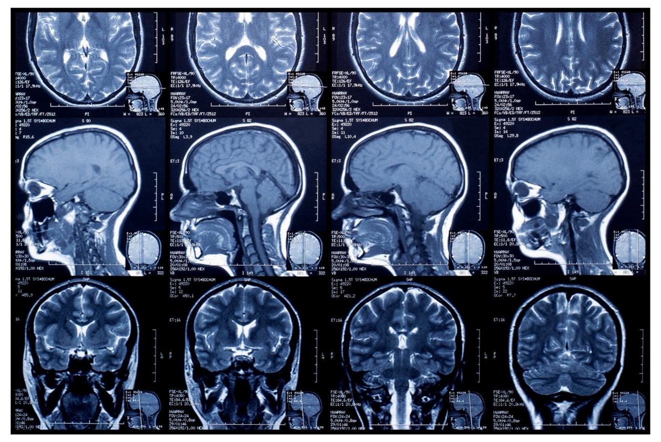
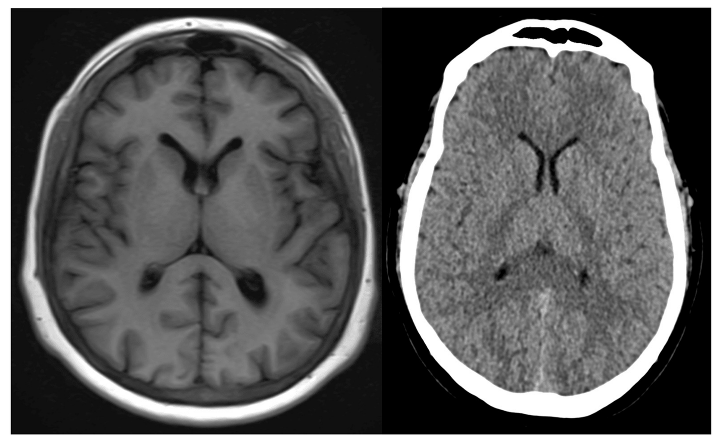
Introduction

- We address the forward scattering problem of calculating the scattered field by an incident plane wave.
- This problem has applications in defense (radar and sonar), medical imaging, oil prospecting, nondestructive testing, and many other fields.
- Find a way to make this problem easier and faster to solve.



MRI Scan
Picture: (News Atlas, 2024)

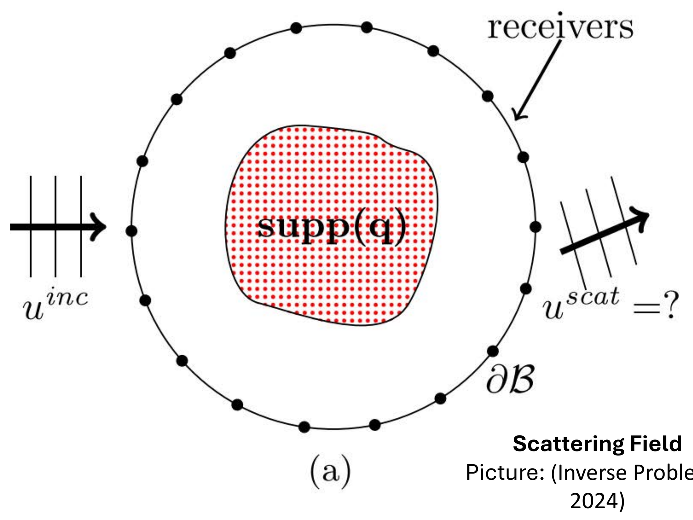


Medical Imaging Scan
Picture: (Mediphany, 2024)

Problem to Solve

- Calculating the scattering field by an incoming incident field u^{inc}.
- It is modeled by the Helmholtz partial differential equation (PDE):

$$\Delta u^{scat} + k^2(1+q)u^{scat} = -k^2 q u^{inc}$$



Scattering Field
Picture: (Inverse Problems, 2024)

Solution

- Using the potential theory formula:

$$u^s = G * e = V_e$$

where G represents Green's Function, we use an integral equation to find e:

$$[I + k^2(1+q)]V_e = -k^2 q u^{inc}$$

And we solve it by using the Nystrom method (The uniform grid + Fourier Transform with BICSTAB preconditioning).

- We implement a fast algorithm to compute the volume potentials of compactly supported functions.
- This algorithm utilizes a window, focusing only on the area of interest.

$$G_k^L(s) = \frac{-1+e^{iLk}(\cos(Ls)-i\frac{k}{s}\sin(Ls))}{(k-s)(k+s)}$$

- This algorithm is $O(k^2 \log(k))$, where k is the wavenumber of the incoming wave.

Objectives

- To improve solution time, we implement a preconditioner to accelerate the iterative method.

$$Ax = b$$
$$M^{-1}Ax = M^{-1}b$$

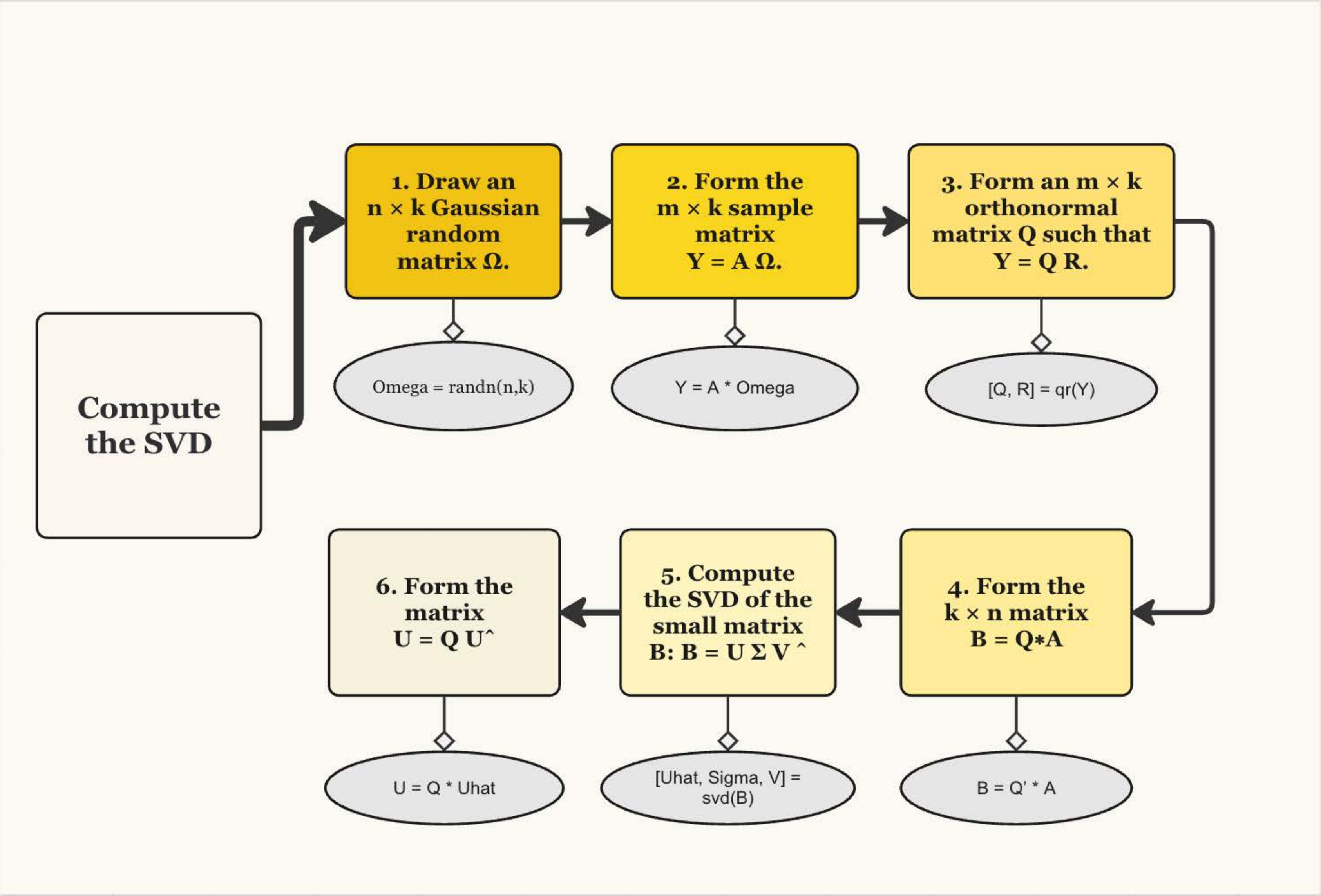
- This method proves to decrease the number of iterations.

Different Strategies

- Domain decomposition, Randomized SVD, Inverse diagonal preconditioning, ILU, and so on.
- We choose Randomized SVD.

Randomized SVD

- We apply a preconditioner to our given matrix: Randomized Singular Value Decomposition.
- We see the speed of solving the matrix increase.
- Randomized SVD is a technique used to approximate the SVD of a given matrix.
- Calculating the SVD of a large matrix can be very computationally expensive.
- Using randomized SVD allows us to efficiently calculate an approximation of the SVD of a matrix.



Results and Conclusion

- Due to time constraints, we were unable to compute the results. Our program did converge on a solution with some speed when testing without a preconditioner. However, after the preconditioner was applied the program never converged.
- While our initial progress was promising, the application of the preconditioner introduced complexities that prevented results within the timeline. Despite these computational challenges, our work establishes a solid baseline for future research about this problem.

Future Work

We intend to study the effect of using domain decomposition-based preconditioners for iteratively solving the discrete system derived from the Lippmann-Schwinger equation. Additionally, we plan to extend our results for 3D problems and other scattering problems, such as the Electromagnetic scattering problems.

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