# Financial Mathematics: Derivative Pricing for Financial Derivatives

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## Introduction

In this project, we researched options. To do this we researched the Black-Scholes Equation which is the most common model to determine the price of an option. The model is:

$$\frac{\partial C}{\partial t} + \frac{1}{2}S^2\frac{\partial^2 C}{\partial S^2} + rS\frac{\partial C}{\partial S} - rC = 0$$

## What is an Option?

- An option is a financial derivative
- The call option allows the buyer the right to buy the stock at a certain price by a certain date
- The put option allows the buyer the right to sell the stock at a certain price by a certain date

## Black-Scholes Solution

Here is the solution of the Black-Scholes Equation:

$$C = N(d_1) * S - K * e^{-rt} * N(d_2)$$
$$P = K * e^{-rt} * N(-d_2) - N(-d_1) * S$$
$$d_1 = \frac{ln(\frac{S}{K}) + (r + 0.5 + \sigma^2) * t}{\sigma * \sqrt{t}}$$
$$d_2 = \frac{ln(\frac{S}{K}) + (r - 0.5 + \sigma^2) * t}{\sigma * \sqrt{t}}$$

**Table 1. Parameters for Black-Scholes Solution**

| | |
|---|---|
| C | Call option |
| P | Put Option |
| N | CDF |
| S | Stock Price |
| K | Strike Price |
| t | time (in years) |
| r | interest rate |
| $\sigma$ | implied volatility (IV) |

- All variables are known except for the IV
- The IV is affected by the time to expiration of the option and the change in stock price
- Using real-time data from different sources, we created graphs that plotted the IV vs time to expiration and the change in stock price vs time to expiration

## Implied Volatility Analysis

### Amazon Option Data and AMC Option Data for change in stock vs expiration time and IV vs expiration time

- We gathered data from many companies to try to back up our claims with evidence
- We analysed certain dates that correlated to important events of the company to see if we could notice any changes
- We looked at expected and unexpected events to see if we could find evidence to support that the IV increases when an event happens, which we did find

## Solving for Implied Volatility

- There are many methods to solve for the IV such as approximation, root finding, and more methods
- I will go over two methods: the Newton-Raphson method and the Bisection method

## Bisection Method

To start we know **f(iv) = BSCall - Price of option**. We want to solve for when **f(iv) = 0** and that will be the volatility. There are four steps to do this:

- Step 1: Pick an upper and lower bound for the volatility that you think is reasonable.
- Step 2: Calculate the middle number between them for the volatility. If f(IV Mid) = 0 then we are done. If not, proceed to the next step.
- Step 3: If f(IV Low)*f(IV Mid)<0 then the root lies between them. However, if f(IV Low) * f(IV Mid) >0 then the root lies between them.
- Step 4: If first condition is true IV Upper = IV Middle. If second condition is true then IV Lower = IV Middle. Then go back to step two.

## Newton-Raphson Method

To start we have:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$
$$f'(x_n) = \textbf{vega}$$

- Vega is the price sensitivity of the option price w.r.t. volatility
- To do this method, you pick a starting point x(0), which is the challenging part. You are finished when f(x(n))=0

## Next Steps/Future Work

With all the data gathered, this will allow us to create different trading strategies

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