QUIZ 3 MAC 2312H (202) Spring 2014 NAME(print) Solution

NO AIDS are permitted for this quiz. **Show all your work.** Correct answers with little or no supporting work will not be given credit. Write legibly. **Circle** your final answer to each problem.

Evaluate the integral.

(1) $\int \tan^{-1} x \, dx$ (Note that "-1" stands for "inverse")

$u = \tan^{-1} x$ $\quad$ $dv = dx$

$du = \frac{1}{1+x^2} dx$ $\quad$ $v = x$

$\int \tan^{-1} x \, dx = x \tan^{-1} x - \int \frac{x}{1+x^2} dx$

For $\int \frac{x}{1+x^2} dx$ $\quad$ $u = 1+2x$

$= \frac{1}{2} \int \frac{1}{u} du$ $\quad$ $du = 2x\, dx$

$= \frac{1}{2} \ln|u| + C$

$= \frac{1}{2} \ln|1+x^2| + C$

So $\int \tan^{-1} x \, dx$

$= \boxed{x \tan^{-1} x - \frac{1}{2} \ln|1+x^2| + C}$

+3

(2) $\int \frac{\ln x}{x^{10}} dx$

$u = \ln x$ $\quad$ $dv = \frac{1}{x^{10}} dx = x^{-10} dx$

$du = \frac{1}{x} dx$ $\quad$ $v = \frac{x^{-10+1}}{-10+1} = -\frac{x^{-9}}{9}$

$$\int \frac{\ln x}{x^{10}} dx = -\frac{x^{-9}}{9} \ln x + \int \frac{1}{x} \cdot \frac{x^{-9}}{9} dx$$

$$= -\frac{x^{-9}}{9} \ln x + \frac{1}{9} \int x^{-10} dx$$

$$= \boxed{-\frac{x^{-9}}{9} \ln x + \frac{1}{9} \cdot \left(\frac{x^{-9}}{-9}\right) + C}$$

+3

(3) $\int e^x \cos x \, dx$

$u = \cos x \qquad dv = e^x dx$

$du = -\sin x \, dx \qquad v = e^x$

(*) $\int e^x \cos x \, dx = e^x \cos x + \int e^x \sin x \, dx$

For $\int e^x \sin x \, dx$

$u = \sin x \qquad dv = e^x dx$

$du = \cos x \, dx \qquad v = e^x$

(**) $\int e^x \sin x \, dx = e^x \sin x - \int e^x \cos x \, dx$

Put (**) into (*)

$$\int e^x \cos x \, dx = e^x \cos x + e^x \sin x - \int e^x \cos x \, dx$$

So $\int e^x \cos x \, dx = \boxed{\frac{1}{2}(e^x \cos x + e^x \sin x) + C}$

+3

(4) $\int \sin^5 x \, dx$

$= \int \sin^4 x \sin x \, dx$

$= \int (\sin^2 x)^2 \sin x \, dx$

$= \int (1 - \cos^2 x)^2 \sin x \, dx$

$= -\int (1-u^2)^2 du \qquad u = \cos x$

$du = -\sin x \, dx$

$= -\int (1 - 2u^2 + u^4) du$

$= -u + 2 \cdot \frac{u^3}{3} - \frac{u^5}{5} + C$

$= \boxed{-\cos x + 2 \cdot \frac{\cos^3 x}{3} - \frac{\cos^5 x}{5} + C}$

+3