QUIZ 5 MAC 2312H (202) Spring 2014 NAME(print) Solution

NO AIDS are permitted for this quiz. **Show all your work**. Correct answers with little or no supporting work will not be given credit. Write legibly. **Circle** your final answer to each problem.

1. Find the limit of the sequence or determine that the limit does not exist.

(1) $\left\{\dfrac{n^3}{n^4+1}\right\}$

$$\lim_{n\to\infty} \frac{n^3}{n^4+1} = \lim_{n\to\infty} \frac{\frac{n^3}{n^4}}{\frac{n^4}{n^4}+\frac{1}{n^4}} = \lim_{n\to\infty} \frac{\frac{1}{n}}{1+\frac{1}{n^4}} = \boxed{0}$$

+2

(2) $\left\{\left(1+\frac{2}{n}\right)^n\right\}$

$$\ln\left(1+\frac{2}{n}\right)^n = n\ln\left(1+\frac{2}{n}\right)$$

$$\lim_{n\to\infty} n\ln\left(1+\frac{2}{n}\right) = \lim_{n\to\infty} \frac{\ln\left(1+\frac{2}{n}\right)}{\frac{1}{n}}$$

$$\overset{L.H}{=} \lim_{n\to\infty} \frac{\frac{1}{1+\frac{2}{n}}\frac{d}{dn}\left(1+\frac{2}{n}\right)}{\frac{d}{dn}\left(\frac{1}{n}\right)}$$

+2

$$= \lim_{n\to\infty} \frac{1}{1+\frac{2}{n}}\cdot 2$$

$$= 2.$$

$$\lim_{n\to\infty}\left(1+\frac{2}{n}\right)^n = e^{\lim_{n\to\infty}\ln\left(1+\frac{2}{n}\right)^n} = \boxed{e^2}$$

+1

2. Find the sum of the series or determine that the series diverges.

(1) $\sum_{n=0}^{\infty} \left(\frac{e}{\pi}\right)^n$

Geometric series with $a = 1$, $r = \frac{e}{\pi}$

+1 +1

Since $|r| = \frac{e}{\pi} < 1$,

The series converges and has sum $\frac{a}{1-r} = \frac{1}{1-\frac{e}{\pi}} = \boxed{\frac{\pi}{\pi - e}}$

+1

(2) $\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)}$

$a_n = \frac{1}{(n+1)(n+2)} = \frac{1}{n+1} - \frac{1}{n+2}$ +1

$S_n = a_1 + a_2 + \cdots + a_n$

$= (\frac{1}{2} - \cancel{\frac{1}{3}}) + (\cancel{\frac{1}{3}} - \cancel{\frac{1}{4}}) + (\cancel{\frac{1}{4}} - \cancel{\frac{1}{5}}) + \cdots + (\cancel{\frac{1}{n}} - \cancel{\frac{1}{n+1}}) - (\cancel{\frac{1}{n+1}} - \frac{1}{n+2})$

$= \frac{1}{2} - \frac{1}{n+2}$ +2

$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \left(\frac{1}{2} - \frac{1}{n+2}\right) = \frac{1}{2}.$

The series converges and has sum $\boxed{\frac{1}{2}}$ +1