QUIZ 1 MAS 3106 (1) Spring 2014 NAME(print) Solution

NO AIDS are permitted except a note with the definition of 'vector space' only. **Show all your work**. Correct answers with little or no supporting work will not be given credit. Write legibly.

1. (6 pts) Let $V = \{(a_1, a_2) : a_1, a_2 \in \mathbb{R}\}$. Define addition of elements of V coordinatewise, and for (a_1, a_2) in V and c in $\mathbb{R}$, define

$$c(a_1, a_2) = \begin{cases} (0,0) & \text{if } c = 0 \\ \left(ca_1, \dfrac{a_2}{c}\right) & \text{if } c \neq 0. \end{cases}$$

Is V a vector space over $\mathbb{R}$ with these operations? Justify your answer.

For any $a, b \in \mathbb{R}$ and $x = (x_1, x_2) \in V$

$$(a+b)(x_1, x_2) = \left((a+b)x_1, \frac{x_2}{a+b}\right)$$

$$a(x_1, x_2) + b(x_1, x_2) = \left(ax_1, \frac{x_2}{a}\right) + \left(bx_1, \frac{x_2}{b}\right)$$

$$= \left((a+b)x_1, \frac{x_2}{a} + \frac{x_2}{b}\right)$$

In general, $\frac{x_2}{a+b} \neq \frac{x_2}{a} + \frac{x_2}{b}$. so $(a+b)x \neq ax + bx$

(For example, with $x_2 = 1$, $a = 2$, $b = 3$,

$$\frac{1}{2+3} \neq \frac{1}{2} + \frac{1}{3}$$)

(VS 8) fails. V is NOT a vector space over $\mathbb{R}$.
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2. (6 pts) Prove that a subset W of a vector space V is a subspace of V if and only if $\mathbf{0} \in W$ and $ax + y \in W$ whenever $a \in F$ and $x, y \in W$.

Proof. " $\Rightarrow$ "

Suppose that W is a subspace of V. Then $\vec{0} \in W$ and for any $a \in F$, $x, y \in W$. $ax \in W$ (W is closed under scalar multiplication) and $ax + y \in W$ (W is closed under addition).

+3

" $\Leftarrow$ "

Since $\vec{0} \in W$, to show that W is a subspace of V, it remains to show that $x + y \in W$ $\forall\, x, y \in W$ and $ax \in W$ $\forall\, a \in F$, $x \in W$.

Since for any $a \in F$, $x, y \in W$, $ax + y \in W$,

we have $x + y = 1 \cdot x + y \in W$

and $ax = ax + \vec{0} \in W$.

+3