**QUIZ 2** MAS 3106 (1) Spring 2014 NAME(print) Solution

**NO AIDS** are permitted for this quiz. **Show all your work.** Correct answers with little or no supporting work will not be given credit. Write legibly.

1. (6 pts) Determine if the following set is a basis for $P_2(\mathbb{R})$.

$$\{1 + 2x - x^2, 4 - 2x + x^2, -1 + 18x - 9x^2\}$$

If $a(1+2x-x^2) + b(4-2x+x^2) + c(-1+18x-9x^2) = \vec{0}$

$(a+4b-c) + (2a-2b+18c)x + (-a+b-9c)x^2 = \vec{0}$ (+2)

then
$$\left.\begin{aligned} a+4b-c&=0 \\ 2a-2b+18c&=0 \\ -a+b-9c&=0 \end{aligned}\right\} \Rightarrow \begin{aligned} a&=-7c \\ b&=2c \end{aligned}$$
(+2)

The given set is linearly dependent. Therefore, it is NOT a basis for $P_2(\mathbb{R})$. (+2)

2. (6 pts) Show that if $S_1$ and $S_2$ are subsets of a vector space $V$ such that $S_1 \subseteq S_2$, then $\text{span}(S_1) \subseteq \text{span}(S_2)$. In particular, if $S_1 \subseteq S_2$ and $\text{span}(S_1) = V$, deduce that $\text{span}(S_2) = V$.

Proof. If $v \in \text{span}(S_1)$, then

$$v = a_1u_1 + a_2u_2 + \cdots + a_nu_n, \qquad u_1, \cdots u_n \in S_1$$
$$a_1 \cdots a_n \in F$$

Since $S_1 \subseteq S_2$, $u_1, \cdots u_n \subseteq S_2$.

+3

So, $v = a_1u_1 + a_2u_2 + \cdots + a_nu_n \in \text{span}(S_2)$.

Therefore, $\text{span}(S_1) \subseteq \text{span}(S_2)$.

If, in addition, $\text{span}(S_1) = V$, we have,

$$V = \text{span}(S_1) \subseteq \text{span}(S_2), \qquad (1)$$

Since $S_2 \subseteq V$ and $V$ is a vector space,

$$\text{span}(S_2) \subseteq V \qquad (2)$$

(1) together with (2),

+3

$$V = \text{span}(S_1) \subseteq \text{span}(S_2) \subseteq V,$$

which implies $\text{span}(S_2) = V$.