QUIZ 3 MAS 3106 (1) Spring 2014 NAME(print) Solution

NO AIDS are permitted for this quiz. **Show all your work.** Correct answers with little or no supporting work will not be given credit. Write legibly.

1. (6 pts) Define $T : \mathbb{R}^2 \to \mathbb{R}^3$ by $T(a_1, a_2) = (a_1 - a_2, a_1, 2a_1 + a_2)$. Let $\alpha = \{(1,2),(2,3)\}$ be an ordered basis for $\mathbb{R}^2$ and $\gamma = \{(1,1,0),(0,1,1),(2,2,3)\}$ be an ordered basis for $\mathbb{R}^3$. Compute $[T]_\alpha^\gamma$.

$$T(1,2) = (1-2, 1, 2+2) = (-1, 1, 4)$$

$$= a(1,1,0) + b(0,1,1) + c(2,2,3)$$

$$\left.\begin{array}{l} a + 2c = -1 \\ a + b + 2c = 1 \\ b + 3c = 4 \end{array}\right\} \Rightarrow a = -\frac{7}{3},\ b = 2,\quad c = \frac{2}{3}$$

$$T(2,3) = (2-3, 2, 4+3) = (-1, 2, 7)$$

$$= a(1,1,0) + b(0,1,1) + c(2,2,3)$$

$$\left.\begin{array}{l} a + 2c = -1 \\ a + b + 2c = 2 \\ b + 3c = 7 \end{array}\right\} \Rightarrow a = -\frac{11}{3}\quad b = 3,\quad c = \frac{4}{3}$$

So $$[T]_\alpha^\gamma = \begin{pmatrix} -\frac{7}{3} & -\frac{11}{3} \\ 2 & 3 \\ \frac{2}{3} & \frac{4}{3} \end{pmatrix}$$

1 pt. for each entry.

2. (6 pts) Let V be a vector space, and let $T : V \to V$ be linear. Prove that $T^2 (= TT) = T_0$ if and only if $R(T) \subseteq N(T)$.

Proof. " $\Longrightarrow$ "

Suppose that $T^2 = T_0$

For any $y \in R(T)$, there is an $x \in V$ such that $T(x) = y$.

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Now, $T(y) = T(T(x)) = T^2(x) = \vec{0}$

So $y \in N(T)$. and hence $R(T) \subseteq N(T)$.

" $\Leftarrow$ "

Suppose that $R(T) \subseteq N(T)$

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For any $x \in V$,

$T^2(x) = T(T(x)) = \vec{0}$ since $T(x) \in R(T) \subseteq N(T)$

So $T^2 = T_0$