QUIZ 5 MAS 3106 (1) Spring 2014 NAME(print) Solution

NO AIDS are permitted for this quiz. **Show all your work**. Correct answers with little or no supporting work will not be given credit. Write legibly.

1. (5 pts) Let $V = \mathbb{R}^2$, $T\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 10a - 6b \\ 17a - 10b \end{pmatrix}$, and $\beta = \left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \end{pmatrix} \right\}$. Determine whether β is a basis consisting of eigenvectors of T.

$$T\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ -3 \end{pmatrix} = 0 \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix} + (-1) \cdot \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$ +2

$$T\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} = 2 \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix} + 0 \cdot \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$ +2

$[T]_\beta = \begin{pmatrix} 0 & 2 \\ -1 & 0 \end{pmatrix}$ is not a diagonal matrix. +1

Hence β does not consist of eigenvectors of T.

2. (7 pts) Given $A = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix} \in M_{2\times 2}(\mathbb{R})$,

(1) Determine all the eigenvalues of A.

$$\det(A - \lambda I) = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}$$

$$= \begin{pmatrix} 1-\lambda & 2 \\ 3 & 2-\lambda \end{pmatrix} = (1-\lambda)(2-\lambda) - 6$$

$$= \lambda^2 - 3\lambda - 4$$

$$= (\lambda - 4)(\lambda + 1)$$

+3

A has eigenvalues $\boxed{\lambda_1 = 4, \quad \lambda_2 = -1}$

(2) For each eigenvalue λ of A, find the set of eigenvectors corresponding to λ.

For $\lambda_1 = 4$. if $(A-4I)x = 0$

$$\left[\begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix} - \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}\right]\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -3 & 2 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

then $-3x_1 + 2x_2 = 0$

+2

set of eigenvectors $\boxed{\left\{ t\begin{pmatrix} 2 \\ 3 \end{pmatrix} : t \in \mathbb{R} \right\}}$

For $\lambda_2 = -1$, if $(A-(-1)I)x = 0$

$$\left[\begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix} - \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\right]\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 2 \\ 3 & 3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

+2

then $2x_1 + 2x_2 = 0$

set of eigenvectors $\boxed{\left\{ t\begin{pmatrix} 1 \\ -1 \end{pmatrix} : t \in \mathbb{R} \right\}}$