QUIZ 6 MAS 3106 (1) Spring 2014 NAME(print) Solution

NO AIDS are permitted for this quiz. **Show all your work.** Correct answers with little or no supporting work will not be given credit. Write legibly.

1. (7 pts) Let $V = \mathrm{P}_3(\mathbb{R})$ and define $T : V \to V$ by $T(f(x)) = f'(x) + f''(x)$. Test T for diagonalizability.

Use the standard ordered basis $\beta = \{1, x, x^2, x^3\}$ for $P_3(\mathbb{R})$

$T(1) = 0$, $T(x) = 1$, $T(x^2) = 2x+2$, $T(x^3) = 3x^2 + 6x$

$$[T]_\beta = \begin{pmatrix} 0 & 1 & 2 & 0 \\ 0 & 0 & 2 & 6 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$ +2

$$C_T(x) = \det([T]_\beta - xI) = \det \begin{pmatrix} -x & 1 & 2 & 0 \\ 0 & -x & 2 & 6 \\ 0 & 0 & -x & 3 \\ 0 & 0 & 0 & -x \end{pmatrix} = x^4$$ splits.

So, T has one eigenvalue $\lambda = 0$ with multiplicity $m = 4$ +2

$$\dim(E_\lambda) = 4 - \operatorname{rank}([T]_\beta - 0 \cdot I) = 4 - \operatorname{rank} \begin{pmatrix} 0 & 1 & 2 & 0 \\ 0 & 0 & 2 & 6 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$= 4 - 3$

$= 1 \neq m = 4.$ +2

So, T is NOT diagonalizable. +1

2. (5 pts) Given $A = \begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix} \in M_{2\times 2}(\mathbb{R})$, test A for diagonalizability.

$$C_A(x) = \det(A - xI) = \det\begin{pmatrix} 1-x & 4 \\ 3 & 2-x \end{pmatrix}$$

$$= (1-x)(2-x) - 12$$

$$= 2 - 3x + x^2 - 12$$

$$= (x-5)(x+2)$$ splits. +2

$\lambda_1 = 5$ with multiplicity $m_1 = 1$, so $\dim(E_{\lambda_1}) = 1$

$\lambda_2 = -2$ with multiplicity $m_2 = 1$, so $\dim(E_{\lambda_2}) = 1$ +2

So, A is diagonalizable. +1