

No aids are permitted, except an approved basic scientific calculator. Show all your work.

Correct answers with little or no supporting work will not be given credit. Write legibly.

1. (15 pts) Find the general solution of the system.

$$X' = \begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix} X$$

$$\text{Let } A = \begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix}. \quad \det(A - \lambda I) = \det \begin{pmatrix} 1-\lambda & 2 \\ 1 & -\lambda \end{pmatrix}$$

$$= \lambda^2 - \lambda - 2$$

$$= (\lambda - 2)(\lambda + 1)$$

(+4)

Eigenvalues $\lambda_1 = 2$, $\lambda_2 = -1$.

$$\text{For } \lambda_1 = 2, \quad (A - 2I) \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} -1 & 2 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$-x + 2y = 0 \Rightarrow x = 2y.$$

$$\text{Choose } v_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

(+4)

$$\text{For } \lambda_2 = -1, \quad (A + I) \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$x + y = 0 \Rightarrow x = -y$$

$$\text{Choose } v_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

(+4)

$$\text{General solution } X(t) = \alpha e^{2t} \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \beta e^{-t} \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

(+3)