MAP 4301(1) Fall 2014 Exam 1 Review Problems

1. Transform the second order equation $x'' + bx' + kx = 0$ into a planar linear system. Find all values of b and k for which the associated coefficient matrix has real distinct eigenvalues and find the general solution of this system.

2. Sketch the bifurcation diagram.

(1) $x' = ax + 3$;

(2) $x' = x(1 - x) - h$.

3. Suppose a species population represented by the function $x(t)$ is harvested at a proportional rate $h \geqslant 0$. It satisfies the differential equation $x'(t) = x(1 - x) - hx$.

(1) Find all equilibrium points.

(2) Determine the nature of each equilibrium point - source or sink.

4. Solve the initial value problem.

(1) $X' = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix} X, \quad X(0) = \begin{pmatrix} 3 \\ \frac{1}{2} \end{pmatrix}$

(2) $X' = \begin{pmatrix} 1 & -2 \\ 3 & -4 \end{pmatrix} X, \quad X(0) = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

5. Find the general solution of the system. Sketch the phase portrait.

(1) $X' = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} X, \quad \lambda_1 < 0 < \lambda_2$

(2) $X' = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} X$

(3) $X' = \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix} X$