MAP 4303(1) Fall 2014   Exam 2 Review Problems

1. For the system $X' = AX$, write out the general solution and sketch the phase portrait.

(1) $A = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$ (2) $A = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix}$ (3) $A = \begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix}$

(4) $A = \begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix}$ (5) $A = \begin{pmatrix} 3 & 2 \\ -2 & 3 \end{pmatrix}$ (6) $A = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$

(7) $A = \begin{pmatrix} -2 & 1 \\ 0 & -2 \end{pmatrix}$

2. For the system $X' = AX$, (a) find the matrix T that puts A in canonical form; (b) find the general solution of both $Y' = (T^{-1}AT)Y$ and $X' = AX$.

(1) $A = \begin{pmatrix} 3 & 2 \\ -1 & 1 \end{pmatrix}$ (2) $A = \begin{pmatrix} 2 & 0 \\ 2 & 2 \end{pmatrix}$ (3) $A = \begin{pmatrix} 1 & 2 \\ 3 & -3 \end{pmatrix}$

3. For the system

$$X' = \begin{pmatrix} 0 & 1 \\ -9 & \beta \end{pmatrix} X$$

where $\beta \in \mathbb{R}$. Identify the type of phase portrait for the three cases: (a) $\beta < 0$ (b) $\beta = 0$ and (c) $\beta > 0$. Give a sketch of the phase portrait for each case.

4. Consider the harmonic oscillator equation $x'' + bx' + kx = 0$, where $b \geqslant 0$ and $k > 0$. Identify the regions in the relevant portion of the bk-plane (b as horizontal axis) where the corresponding system has similar phase portraits.