QUIZ 1 MAP 4303 (1) Fall 2014 NAME (print) Solution

No aids are permitted, except an approved basic scientific calculator. **Show all your work.** Correct answers with little or no supporting work will not be given credit. Write legibly.

1. (6 pts) Find all equilibrium solutions and determine whether they are sinks, sources, or neither. Sketch the phase line.

$$x' = x^4 - x^2$$

$x' = x^4 - x^2 = x^2(x+1)(x-1) = 0$

Equilibrium solutions

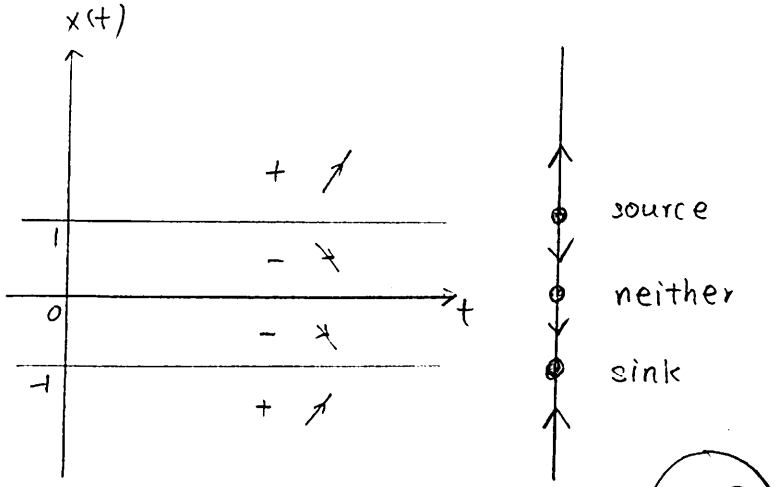
$x = 0,\ x = 1,\ x = -1$

When $x < -1$, $x' > 0$

$-1 < x < 0$, $x' < 0$

$0 < x < 1$, $x' < 0$

$x > 1$, $x' > 0$



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2. (9 pts) Sketch the bifurcation diagram of the family of differential equations depending on a parameter a.

$$x' = x^2 - ax$$

When $a = 0$, $x' = x^2 > 0$ except at equilibrium solution $x = 0$.

When $a \neq 0$, $x' = x(x-a)$.

equilibrium solutions $x = 0,\ x = a$,

When $a < 0$, $x' > 0$ if $x < a$

$x' < 0$ if $a < x < 0$

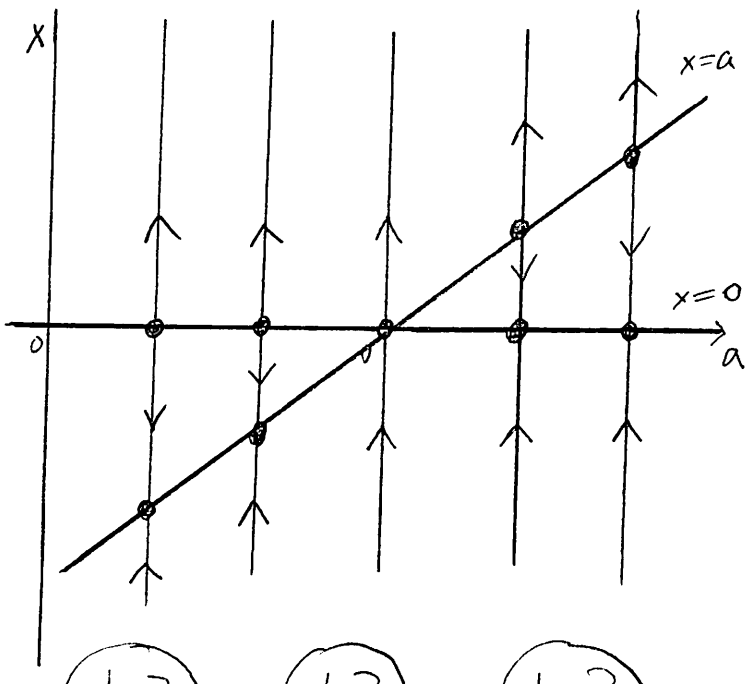
$x' > 0$ if $x > a$

When $a > 0$, $x' > 0$ if $x < 0$

$x' < 0$ if $0 < x < a$

$x' > 0$ if $x > a$

Bifurcation at $a = 0$



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