QUIZ 4 MAP 4303 (1) Fall 2014 NAME (print) Solution

No aids are permitted, except an approved basic scientific calculator. **Show all your work.** Correct answers with little or no supporting work will not be given credit. Write legibly.

1. For the system $X' = AX$, find the matrix T that puts A in canonical form.

(1) $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$\det(A - \lambda I) = \det\begin{pmatrix} -\lambda & 1 \\ 1 & -\lambda \end{pmatrix} = \lambda^2 - 1.$

Eigenvalues $\lambda_1 = 1$, $\lambda_2 = -1$ +2

$\lambda_1 = 1$: $\begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ choose $V_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ +2

$\lambda_2 = -1$: $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ choose $V_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$\boxed{T = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}}$ +3

(note that one can also choose $V_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$, then $T = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$)

(2) $A = \begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}$

$\det(A - \lambda I) = \det\begin{pmatrix} 1-\lambda & 1 \\ -1 & -\lambda \end{pmatrix} = \lambda^2 - \lambda + 1$, $\frac{1 \pm \sqrt{3}i}{2}$ +2

$\lambda = \frac{1+\sqrt{3}i}{2}$, $\begin{pmatrix} 1 - \frac{1+\sqrt{3}i}{2} & 1 \\ -1 & -\frac{1+\sqrt{3}i}{2} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$

$\frac{1-\sqrt{3}i}{2}x + y = 0$. $y = -\frac{1-\sqrt{3}i}{2}x = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$

$V = \begin{pmatrix} 1 + 0i \\ -\frac{1}{2} + \frac{\sqrt{3}}{2}i \end{pmatrix} = \begin{pmatrix} 1 \\ -\frac{1}{2} \end{pmatrix} + i\begin{pmatrix} 0 \\ \frac{\sqrt{3}}{2} \end{pmatrix}$ +3

$\boxed{T = \begin{pmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}}$ +3