QUIZ 6 MAP 4303 (1) Fall 2014 NAME (print) Solution

No aids are permitted, except an approved basic scientific calculator. **Show all your work.** Correct answers with little or no supporting work will not be given credit. Write legibly.

Find the general solution of the system $X' = AX$, where

$$A = \begin{pmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 1 \\ 0 & 0 & \lambda \end{pmatrix}$$

Let $X(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix}$. $\begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} = \begin{pmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 1 \\ 0 & 0 & \lambda \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

$$x_1' = \lambda x_1 + x_2 \quad (1)$$
$$x_2' = \lambda x_2 + x_3 \quad (2)$$
$$x_3' = \lambda x_3 \quad (3)$$

From (3). $\boxed{x_3(t) = C_3 e^{\lambda t}}$ +4

Then $x_2' = \lambda x_2 + C_3 e^{\lambda t}$ (4)

Assume $x_2(t) = C_2 e^{\lambda t} + \alpha t e^{\lambda t}$ (5)

Substituting (5) into (4)

$$\underline{C_2 \lambda e^{\lambda t}} + \alpha e^{\lambda t} + \alpha t \lambda e^{\lambda t} = \underline{\lambda C_2 e^{\lambda t}} + \lambda \alpha t e^{\lambda t} + C_3 e^{\lambda t}$$

implies that $\alpha = C_3$ So $\boxed{x_2(t) = C_2 e^{\lambda t} + C_3 t e^{\lambda t}}$ +4

Then $x_1' = \lambda x_1 + C_2 e^{\lambda t} + C_3 t e^{\lambda t}$ (6)

Assume $x_1(t) = C_1 e^{\lambda t} + \alpha t e^{\lambda t} + \beta t^2 e^{\lambda t}$ (7)

Substituting (7) into (6)

$$\underline{C_1 \lambda e^{\lambda t}} + \alpha e^{\lambda t} + \alpha t \lambda e^{\lambda t} + \beta 2 t e^{\lambda t} + \beta t^2 \lambda e^{\lambda t} = \underline{\lambda C_1 e^{\lambda t}} + \lambda \alpha t e^{\lambda t} + \lambda \beta t^2 e^{\lambda t} + C_2 e^{\lambda t} + C_3 t e^{\lambda t}.$$

implies that $\alpha = C_2$, $\beta = \frac{C_3}{2}$ +4

So $\boxed{x_1(t) = C_1 e^{\lambda t} + C_2 t e^{\lambda t} + C_3 \frac{t^2}{2} e^{\lambda t}}$

$$X(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix} = C_1 e^{\lambda t}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + C_2 e^{\lambda t}\begin{pmatrix} t \\ 1 \\ 0 \end{pmatrix} + C_3 e^{\lambda t}\begin{pmatrix} \frac{t^2}{2} \\ t \\ 1 \end{pmatrix}$$ +3