

No aids are permitted, except an approved basic scientific calculator. Show all your work. Correct answers with little or no supporting work will not be given credit. Write legibly.

Find all equilibrium points of the nonlinear system. Then determine the type of each equilibrium point.

$$x' = x^2 - 3xy + 2x, \quad y' = x + y - 1$$

Let $x^2 - 3xy + 2x = 0$ (1)

$x + y - 1 = 0$ (2)

From (2) $y = 1 - x$

Plug into (1) $x^2 - 3x(1-x) + 2x = 0$

$$4x^2 - x = 0$$

$$x(4x - 1) = 0 \quad x = 0, \frac{1}{4}$$

When $x = 0$, $y = 1$; when $x = \frac{1}{4}$, $y = \frac{3}{4}$

Equilibrium points: $(0, 1)$, $(\frac{1}{4}, \frac{3}{4})$

$$J(x, y) = \begin{pmatrix} 2x - 3y + 2 & -3x \\ 1 & 1 \end{pmatrix}$$

For $(0, 1)$:

saddle

$$J(0, 1) = \begin{pmatrix} -1 & 0 \\ 1 & 1 \end{pmatrix}$$

Eigenvalues $-1, 1$

For $(\frac{1}{4}, \frac{3}{4})$:

spiral source

$$J(\frac{1}{4}, \frac{3}{4}) = \begin{pmatrix} \frac{1}{4} & -\frac{3}{4} \\ 1 & 1 \end{pmatrix}$$

$$(\frac{1}{4} - \lambda)(1 - \lambda) + \frac{3}{4} = 0$$

$$\lambda^2 - \frac{5}{4}\lambda + 1 = 0$$

$$4\lambda^2 - 5\lambda + 4 = 0$$

Eigenvalues $\frac{5}{8} \pm \frac{\sqrt{39}}{8}i$

(+5)

(+5)

(+5)