QUIZ 2 MAS 3106 (1) Spring 2015 NAME (print) Solution

No aids are permitted. **Show all your work.** Correct answers with little or no supporting work will not be given credit. Write legibly.

1. (7 pts) Determine if the vectors are linearly independent.

$$\begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 7 \\ 2 \\ -6 \end{bmatrix}, \begin{bmatrix} 9 \\ 4 \\ -8 \end{bmatrix}$$

If $x_1 \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 7 \\ 2 \\ -6 \end{bmatrix} + x_3 \begin{bmatrix} 9 \\ 4 \\ -8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $(*)$,

augmented matrix $\begin{bmatrix} 5 & 7 & 9 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & -6 & -8 & 0 \end{bmatrix} \xrightarrow{3R2+R3} \begin{bmatrix} 5 & 7 & 9 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & 0 & 4 & 0 \end{bmatrix}$ +4

At this point, it is clear that there are three basic variables and no free variables.

So the equation $(*)$ has only the trivial solution, namely $x_1 = x_2 = x_3 = 0$, and the given vectors are linearly independent. +3

2. (8 pts) Let $\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\mathbf{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\mathbf{y}_1 = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$, $\mathbf{y}_2 = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$, and let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation that maps $\mathbf{e}_1$ into $\mathbf{y}_1$ and maps $\mathbf{e}_2$ into $\mathbf{y}_2$. Find the images of $\begin{bmatrix} 5 \\ -3 \end{bmatrix}$ and $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$.

$$\begin{aligned} T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) &= T\left(x_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = T(x_1 \vec{e}_1 + x_2 \vec{e}_2) \\ &= x_1 T(\vec{e}_1) + x_2 T(\vec{e}_2) \\ &= x_1 \begin{bmatrix} 2 \\ 5 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 6 \end{bmatrix} = \begin{bmatrix} 2x_1 - x_2 \\ 5x_1 + 6x_2 \end{bmatrix}. \end{aligned}$$

+4

$$T\left(\begin{bmatrix} 5 \\ -3 \end{bmatrix}\right) = \begin{bmatrix} 2 \cdot 5 - (-3) \\ 5 \cdot 5 + 6 \cdot (-3) \end{bmatrix} = \begin{bmatrix} 13 \\ 7 \end{bmatrix}$$

+4