

No aids are permitted. Show all your work. Correct answers with little or no supporting work will not be given credit. Write legibly.

1. (6 pts) Compute the determinant.

$$\begin{vmatrix} 6 & 0 & 0 & 5 \\ 1 & 7 & 2 & -5 \\ 2 & 0 & 0 & 0 \\ 8 & 3 & 1 & 8 \end{vmatrix}$$

$$= (-1)^{3+1} \cdot 2 \cdot \begin{vmatrix} 0 & 0 & 5 \\ 7 & 2 & -5 \\ 3 & 1 & 8 \end{vmatrix}$$

(+3)

$$= 2 \cdot (-1)^{1+3} \cdot 5 \begin{vmatrix} 7 & 2 \\ 3 & 1 \end{vmatrix}$$

$$= 2 \cdot 5 \cdot 1$$

$$= 10$$

(+3)

2. (9 pts) Find an explicit description of $\text{Nul}A$, by listing vectors that span the null space.

$$A = \begin{bmatrix} 1 & 2 & 4 & 0 \\ 0 & 1 & 3 & -2 \end{bmatrix}$$

$$\text{Solve } \begin{bmatrix} 1 & 2 & 4 & 0 \\ 0 & 1 & 3 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 4 & 0 & 0 \\ 0 & 1 & 3 & -2 & 0 \end{bmatrix} \xrightarrow{(-2)R_2 + R_1} \begin{bmatrix} 1 & 0 & -2 & 4 & 0 \\ 0 & 1 & 3 & -2 & 0 \end{bmatrix}$$

$$\begin{cases} x_1 = 2x_3 - 4x_4 \\ x_2 = -3x_3 + 2x_4 \\ x_3 \text{ free} \\ x_4 \text{ free} \end{cases}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2x_3 - 4x_4 \\ -3x_3 + 2x_4 \\ x_3 \\ x_4 \end{bmatrix} = x_3 \begin{bmatrix} 2 \\ -3 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -4 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

(+6)

$$\text{Nul}A = \text{Span} \left\{ \begin{bmatrix} 2 \\ -3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\}$$

(+3)