

No aids are permitted. Show all your work. Correct answers with little or no supporting work will not be given credit. Write legibly.

1. (7 pts) The set $\mathcal{B} = \{1 + t^2, t + t^2, 1 + 2t + t^2\}$ is a basis for $\mathbb{P}_2$. Find the coordinate vector of $p(t) = 1 + 4t + 7t^2$ relative to $\mathcal{B}$.

$$1 + 4t + 7t^2 = c_1(1 + t^2) + c_2(t + t^2) + c_3(1 + 2t + t^2)$$

$$1 + 4t + 7t^2 = (c_1 + c_3) + (c_2 + 2c_3)t + (c_1 + c_2 + c_3)t^2$$

$$1 = c_1 + c_3$$

$$\Rightarrow 4 = c_2 + 2c_3$$

$$7 = c_1 + c_2 + c_3$$

(+5)

$$\Rightarrow c_1 = 2$$

$$c_2 = 6$$

$$c_3 = -1$$

$$\Rightarrow [p(t)]_{\mathcal{B}} = \begin{bmatrix} 2 \\ 6 \\ -1 \end{bmatrix}$$

(+2)

2. (8 pts) Determine the dimensions of $\text{Nul}A$ and $\text{Col}A$.

$$A = \begin{bmatrix} \boxed{1} & 2 & 3 & 0 & 0 \\ 0 & 0 & \boxed{1} & 0 & 1 \\ 0 & 0 & 0 & \boxed{1} & 0 \end{bmatrix}$$

$$\dim \text{Nul}A = \text{the number of free variables} = 2$$

(+4)

$$\dim \text{Col}A = \text{the number of pivot columns} = 3$$

(+4)