

No aids are permitted. Show all your work. Correct answers with little or no supporting work will not be given credit. Write legibly.

1. (9 pts) Find a basis for the eigenspace corresponding to the eigenvalue.

$$A = \begin{bmatrix} -4 & 1 & 1 \\ 2 & -3 & 2 \\ 3 & 3 & -2 \end{bmatrix}, \quad \lambda = -5$$

Solve  $[A - \lambda I] \vec{x} = \vec{0}$ .

$$A - (-5)I = \begin{bmatrix} -4+5 & 1 & 1 \\ 2 & -3+5 & 2 \\ 3 & 3 & -2+5 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_2 \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

(+5)

Basis  $\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\}$

(+4)

2. (6 pts) Find the characteristic polynomial and the eigenvalues of the matrix.

$$A = \begin{bmatrix} -4 & 2 \\ 6 & 7 \end{bmatrix}$$

$$\det(A - \lambda I) = \det \begin{bmatrix} -4-\lambda & 2 \\ 6 & 7-\lambda \end{bmatrix} = (-4-\lambda)(7-\lambda) - 12$$

$$= \lambda^2 - 3\lambda - 40$$

$$= (\lambda - 8)(\lambda + 5)$$

(+3)

Eigenvalues 8, -5.

(+3)