QUIZ 6 MAS 3106 (1) Spring 2015 NAME (print) Solution

No aids are permitted. **Show all your work.** Correct answers with little or no supporting work will not be given credit. Write legibly.

1. (7 pts) Diagonalize the matrix, if possible.

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 4 \end{bmatrix}$$

$\det(A - \lambda I) = \det \begin{bmatrix} 2-\lambda & -1 \\ 1 & 4-\lambda \end{bmatrix} = (2-\lambda)(4-\lambda)+1 = \lambda^2 - 6\lambda + 9 = (\lambda-3)^2$

Eigenvalue $\lambda = 3$ with multiplicity 2 (+3)

Solve $(A - 3\lambda)\vec{x} = \vec{0}$. $\left[\begin{array}{cc|c} -1 & -1 & 0 \\ 1 & 1 & 0 \end{array}\right]$ $x_1 = -x_2$, x_2 is free

The matrix A is not diagonalizable because A does not have two linearly independent eigenvectors (the dimension of the eigenspace corresponding to $\lambda=3$ is 1, which is not equal to the multiplicity of λ) (+4)

2. (8 pts) Find the complex eigenvalues and a basis for each eigenspace.

$$A = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$$

$\det(A - \lambda I) = \det \begin{bmatrix} 1-\lambda & -2 \\ 1 & 3-\lambda \end{bmatrix} = (1-\lambda)(3-\lambda)+2 = \lambda^2 - 4\lambda + 5$ (+2)

$\frac{4 \pm \sqrt{16-20}}{2} = \boxed{2 \pm i}$

$\lambda_1 = 2 + i$

Solve $[A - (2+i)I]\vec{x} = \vec{0}$

$\left[\begin{array}{cc|c} -1-i & -2 & 0 \\ 1 & 1-i & 0 \end{array}\right]$ $x_1 = -(1-i)x_2$ $\boxed{\vec{v_1} = \begin{bmatrix} -1+i \\ 1 \end{bmatrix}}$ (+3)

$\lambda_2 = 2 - i$, $\boxed{v_2 = \begin{bmatrix} -1-i \\ 1 \end{bmatrix}}$ (+3)