No aids are permitted. **Show all your work.** Correct answers with little or no supporting work will not be given credit. Write legibly.

Let $y = \begin{bmatrix} 5 \\ -9 \\ 5 \end{bmatrix}$, $u_1 = \begin{bmatrix} -3 \\ -5 \\ 1 \end{bmatrix}$, $u_2 = \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix}$. Find the distance from y to the plane in $\mathbb{R}^3$ spanned by u_1 and u_2.

Let $W = \text{span}\ \{\vec{u_1}, \vec{u_2}\}$

$$\hat{y} = \text{proj}_W \vec{y} = \text{proj}_{\vec{u_1}} \vec{y} + \text{proj}_{\vec{u_2}} \vec{y}$$

$$= \frac{\vec{y} \cdot \vec{u_1}}{\vec{u_1} \cdot \vec{u_1}} \vec{u_1} + \frac{\vec{y} \cdot \vec{u_2}}{\vec{u_2} \cdot \vec{u_2}} \vec{u_2}$$

$$= \frac{5(-3) + (-9)(-5) + 5 \cdot 1}{(-3)(-3) + (-5)(-5) + 1 \cdot 1} \begin{bmatrix} -3 \\ -5 \\ 1 \end{bmatrix} + \frac{5(-3) + (-9)2 + 5 \cdot 1}{(-3)(-3) + 2 \cdot 2 + 1 \cdot 1} \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix}$$

$$= \frac{35}{35} \begin{bmatrix} -3 \\ -5 \\ 1 \end{bmatrix} + \frac{-28}{14} \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -9 \\ -1 \end{bmatrix} \qquad \boxed{+10}$$

The distance from $\vec{y}$ to W is $\|\vec{y} - \hat{y}\|$

$$\vec{y} - \hat{y} = \begin{bmatrix} 5 \\ -9 \\ 5 \end{bmatrix} - \begin{bmatrix} 3 \\ -9 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 6 \end{bmatrix}$$

$$\|\vec{y} - \hat{y}\| = \sqrt{2 \cdot 2 + 0 \cdot 0 + 6 \cdot 6} \quad \boxed{= \sqrt{40} = 2\sqrt{10}} \qquad \boxed{+5}$$