Erratum to: "Extending the Type Reproduction Number to Infectious Disease Control Targeting Contacts Between Types"

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An error in the definition of the target reproduction number $\mathcal{T}_S$ with respect to the target set S (see Eq. (2.1) in [4]) was recently discovered. That definition is correct if the target set S consists of any number of entries from only one row (or column) of the $n \times n$ next-generation matrix K, or all nonzero entries from several rows (or columns) of K. However, it is not correct for a general target set S, for example, $\mathcal{T}_U, \mathcal{T}_V$ in Section 3.1, and two values of $\mathcal{T}_S$ in Section 5.4 of [4], although the stated formulas hold. A correct definition of the target reproduction number $\mathcal{T}_S$ is as follows.

Let $K = [k_{ij}]$ be a nonnegative irreducible $n \times n$ (next-generation) matrix. Suppose that several entries of K are subject to change (target), either a decrease or increase. Let S be the *target set* consisting of all entries subject to change, and K_S be the *target matrix* corresponding to the target set S defined as follows: $[K_S]_{ij} = k_{ij}$ if $(i, j) \in S$ and 0 otherwise. If $\rho(K - K_S) < 1$, then the *target reproduction number* $\mathcal{T}_S$ is defined as

$$\mathcal{T}_S = \rho(K_S(I - K + K_S)^{-1}), \tag{1}$$

where I is the $n \times n$ identity matrix and ρ denotes the spectral radius of the matrix.

In [4], it was misstated that the target matrix K_S can be written as $P_{S_1} K P_{S_2}$, which thus leads to the incorrect definition as in Eq. (2.1) in [4]. Since all derivations and calculations in [4] utilize the correct target matrix K_S (although stated in the form of $P_{S_1} K P_{S_2}$), these results and examples in [4] remain correct, except for the typos stated below. In all places in [4], $P_{S_1}KP_{S_2}$ should be replaced by K_S. For example, it is stated in the proof of Theorem 2.2 in [4] that "each targeted entry of K appears only in the term $P_{S_1}KP_{S_2}$"; here $P_{S_1}KP_{S_2}$ should be replaced by the target matrix K_S.

When the target set S consists of one or more rows and all n columns of K, the target reproduction number becomes the type reproduction number, previously defined in [1, 3]. In particular, when $S = \{(i,j)|1 \leq j \leq n\}$ for some i (namely, targeting all entries in the i-th row of K), the target reproduction number becomes the type reproduction number denoted by $\mathcal{T}_i$ [1, 3]. When $S = \{(j,i)|1 \leq j \leq n\}$ for some i (namely, targeting all entries in the i-th column), the target reproduction number is also equal to $\mathcal{T}_i$ as shown in [2, Section 5]. In general, K_S can consist of entries from the same row or from different rows in K; see [4, Section 5]. If $S = \{(i,j)\}$ (namely, only one entry is targeted), then the target reproduction number is customarily denoted by $\mathcal{T}_{ij}$. The formulas for $\mathcal{T}_{ij}$ and $\mathcal{T}_i$ stated in Eq. (2.2) and Eq. (2.3) of [4] are correct. In addition, new expressions for $\mathcal{T}_{ij}$ and $\mathcal{T}_i$ are derived in [2, Theorems 4.1 and 5.3] in terms of cycles in the weighted digraph associated with K.

The following typos and inserts for clarification in [4] should be noted:

- Page 1071, line -1: the $(1,2)$ entry in the matrix is $k_{12}/(1-k_{22})$ rather than 0, but the stated result for $\mathcal{T}_S$ holds.
- Page 1072, line -1: the coefficient for the product $k_{11}k_{12}k_{21}k_{22}$ in $\mathcal{T}_V$ should be 4 rather than 1.
- Page 1073, line -11: insert "and this arc is the last arc of the walk" before the end of the sentence.
- Page 1073, line -1: insert "and this arc is the last arc of the walk" before "but no other arcs".
- Page 1074, line 7: insert "and this arc is the last arc of the walk" before the end of the sentence.

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References

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