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Note

Gallai–Ramsey numbers of C_7 with multiple colors

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ABSTRACT

We study Ramsey-type problems in Gallai-colorings. Given a graph H and an integer $k \geq 1$, the Gallai–Ramsey number $GR_k(H)$ is the least positive integer n such that every k-coloring of the edges of the complete graph on n vertices contains either a rainbow triangle or a monochromatic copy of H. It turns out that $GR_k(H)$ behaves more nicely than the classic Ramsey number $R_k(H)$. However, finding exact values of $GR_k(H)$ is far from trivial. In this paper, we prove that $GR_k(C_7) = 3 \cdot 2^k + 1$ for all $k \geq 1$. Our technique relies heavily on the structural result of Gallai on edge-colorings of complete graphs without rainbow triangles.

1. Introduction

In this paper, we only consider finite simple graphs. The complete graph and the cycle on n vertices are denoted K_n and C_n, respectively. We use $|G|$ to denote the number of vertices of a graph G.

For an integer $k \geq 1$, let $c : E(G) \to [k]$ be a k-edge-coloring of a complete graph G, where $[k] := \{1, 2, \ldots, k\}$. Then c is a *Gallai-coloring* of G if G contains no rainbow triangle (that is, a triangle with all its edges different colors) under c. Gallai-colorings naturally arise in several areas including in information theory [14], in the study of partially ordered sets, as in Gallai's original paper [10], and in the study of perfect graphs [4]. There are now a variety of papers which consider Ramsey-type problems in Gallai-colorings (see, e.g., [5,8,11–13]). These works mainly focus on finding various monochromatic subgraphs in such colorings. More information on this topic can be found in [7,9].

For a graph G and a set $A \subseteq V(G)$, we use $G[A]$ to denote the subgraph of G obtained from G by deleting all vertices in $V(G)\backslash A$. A graph H is an *induced subgraph* of G if $H = G[A]$ for some $A \subseteq V(G)$. Recall that the classical Ramsey number $R_k(H)$ of a graph H is the least positive integer n such that every k-edge-coloring of K_n contains a monochromatic copy of H. Ramsey numbers are notoriously difficult to compute in general. In this paper, we consider Gallai–Ramsey problems. Given a graph H and an integer $k \geq 1$, the *Gallai–Ramsey number* $GR_k(H)$ is the least positive integer n such that every k-edge-coloring of K_n contains either a rainbow triangle or a monochromatic copy of H. Clearly, $GR_k(H) \leq R_k(H)$. We will utilize the following important structural result of Gallai [10] on Gallai-colorings of complete graphs.

Theorem 1.1 ([10]). *For any Gallai-coloring c of a complete graph G with $|G| \geq 2$, $V(G)$ can be partitioned into nonempty sets $V_1, V_2, \ldots, V_p$ with $p > 1$ so that at most two colors are used on the edges in $E(G)\backslash(E(V_1) \cup \cdots \cup E(V_p))$ and only one color is used on the edges between any fixed pair (V_i, V_j) under c, where $E(V_i)$ denotes the set of edges in $G[V_i]$ for all $i \in [p]$.*

The partition given in Theorem 1.1 is called a *Gallai-partition* of G under c. Given a Gallai-partition $V_1, V_2, \ldots, V_p$ of the complete graph G under c, let $v_i \in V_i$ for all $i \in [p]$ and let $\mathcal{R} := G[\{v_1, v_2, \ldots, v_p\}]$. Then $\mathcal{R}$ is the *reduced graph* of G

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corresponding to the given Gallai-partition under c. Clearly, $\mathcal{R}$ is isomorphic to K_p. By Theorem 1.1, all edges in $\mathcal{R}$ are colored by at most two colors under c. One can see that any monochromatic H in $\mathcal{R}$ under c will result in a monochromatic H in G under c. It is not a surprise then that Gallai–Ramsey numbers $GR_k(H)$ are related to the classical Ramsey numbers $R_2(H)$. Recently, Fox, Grinshpun and Pach posed the following conjecture on $GR_k(H)$ when H is a complete graph.

Conjecture 1.2 ([7]). *For all $k \geq 1$ and $t \geq 3$,*

$$GR_k(K_t) = \begin{cases} (R_2(K_t) - 1)^{k/2} + 1 & \text{if } k \text{ is even} \\ (t-1)(R_2(K_t) - 1)^{(k-1)/2} + 1 & \text{if } k \text{ is odd.} \end{cases}$$

The first case of Conjecture 1.2 was verified in 1983 due to Chung and Graham [5]. The next open case when $t = 4$ was recently settled in [15]. A simpler proof of Theorem 1.3 can be found in [12].

Theorem 1.3 ([5]). *For all $k \geq 1$,* $GR_k(C_3) = \begin{cases} 5^{k/2} + 1 & \text{if } k \text{ is even} \\ 2 \cdot 5^{(k-1)/2} + 1 & \text{if } k \text{ is odd.} \end{cases}$

The following is a result in [12] on the general behavior of $GR_k(H)$.

Theorem 1.4 ([12]). *Let H be a fixed graph with no isolated vertices and let $k \geq 1$ be an integer. Then $GR_k(H)$ is exponential in k if H is not bipartite, linear in k if H is bipartite but not a star, and constant (does not depend on k) when H is a star.*

It turns out that for some graphs H (e.g., when $H = C_3$), $GR_k(H)$ behaves nicely, while the order of magnitude of $R_k(H)$ seems hopelessly difficult to determine. It is worth noting that finding exact values of $GR_k(H)$ is far from trivial, even when $|H|$ is small. Theorem 1.5 is a result of Fujita and Magnant [8], which provides a lower bound for $GR_k(C_{2n+1})$ and exact values of $GR_k(C_5)$.

Theorem 1.5 ([8]). *For all $k \geq 1$ and $n \geq 2$,*

(a) $GR_k(C_{2n+1}) \geq n \cdot 2^k + 1$,
(b) $GR_k(C_5) = 2^{k+1} + 1$.

In this paper, we determine the Gallai–Ramsey numbers for C_7. We state and prove our main result in Section 2. Very recently, Gallai–Ramsey numbers of odd cycles of length at least 9 have been completely settled by Bosse and Song [2] for C_9 and C_{11}, Bosse, Song and Zhang [3] for C_{13} and C_{15}, and Zhang, Chen and Song [16] for all odd cycles of length at least 17. We shall need the following two results in our proof of Gallai–Ramsey numbers of C_7.

Theorem 1.6 ([1]). *For all $n \geq 2$, $R_2(C_{2n+1}) = 4n + 1$.*

Theorem 1.7 ([6]). *$R_3(C_7) = 25$.*

Finally, we need to introduce more notation. For positive integers n, k and $G = K_n$, let c be any k-edge-coloring of G with color classes $E_1, \ldots, E_k$. Then c is *bad* if G contains neither a rainbow K_3 nor a monochromatic C_7 under c. For any $E \subset E(G)$, let $G[E]$ denote the subgraph of G with vertex set $V(E)$ and edge set E. Let H be an induced subgraph of G and let $E = E_i \cap E(H)$ for some $i \in [k]$. Then $G[E]$ is an *induced star* in H if $G[E]$ is isomorphic to $K_{1,|V(E)|}$; and $G[E]$ is an *induced matching* in H if E is a matching in H. For two disjoint sets $A, B \subseteq V(G)$, if all the edges between A and B in G are colored the same color under c, say, blue, we say that A is *blue-complete* to B.

2. Main result

We are now ready to prove our main result below.

Theorem 2.1. *For all $k \geq 1$, $GR_k(C_7) = 3 \cdot 2^k + 1$.*

Proof. By Theorem 1.5(a), $GR_k(C_7) \geq 3 \cdot 2^k + 1$ for all $k \geq 1$. We next show that $GR_k(C_7) \leq 3 \cdot 2^k + 1$ for all $k \geq 1$. Clearly, $GR_1(C_7) \leq 7$. By Theorems 1.6 and 1.7, $GR_2(C_7) \leq R_2(C_7) = 13$ and $GR_3(C_7) \leq R_3(C_7) = 25$. Therefore, $GR_k(C_7) \leq 3 \cdot 2^k + 1$ for all $k \in \{1, 2, 3\}$. So we may assume that $k \geq 4$. Let $G = K_{3 \cdot 2^k + 1}$ and let c be any k-edge-coloring of G such that G admits no rainbow triangle. We next show that G contains a monochromatic C_7 under c.

Suppose that G does not contain a monochromatic C_7 under c. Then c is bad. Among all complete graphs on $3 \cdot 2^k + 1$ vertices with a bad k-edge-coloring, we choose G with k minimum. We next prove a series of claims.

Claim 1. *For any induced subgraph H of G, if there exist two vertices, say, u and v, in $V(G \backslash H)$, such that all edges between $\{u, v\}$ and $V(H)$ are colored the same color, say blue, then $|H| \leq 3 \cdot 2^{k-1-q} + 2^{k-q}$, where $0 \leq q \leq k-1$ is the number of colors missing on the edges of H under c, other than blue.*

Proof. Suppose that $|H| \geq 3 \cdot 2^{k-1-q} + 2^{k-q} + 1$. Let E be the set of all blue edges in H. Then $q \leq k-2$, otherwise we obtain a blue C_7 in $G[V(H) \cup \{u, v\}]$, a contradiction. If $|V(E)| \leq 4$ or $G[E]$ is an induced star with its center vertex, say w, in H, then $H \backslash A$ has no blue edges and the edges of $H \backslash A$ are colored by at most $k-1-q$ colors under c, where $A = V(E)$ when $|V(E)| \leq 4$ and $A = \{w\}$ when $G[E]$ is an induced star. Clearly, $|A| \leq 4$ and $|H \backslash A| \geq 3 \cdot 2^{k-1-q} + 2^{k-q} + 1 - 4 \geq 3 \cdot 2^{k-1-q} + 1$. By the choice of k, $H \backslash A$ has a monochromatic C_7, a contradiction. Thus $|V(E)| \geq 5$ and E is not an induced star in H.

Next suppose E is an induced matching in H. Let $E := \{u_1v_1, u_2v_2, \ldots, u_{|E|}v_{|E|}\}$, and let $U := \{u_1, u_2, \ldots, u_{|E|}\}$. Since E is an induced matching in H, we see that $H \backslash U$ has no blue edges, and so the edges of $H \backslash U$ are colored by at most $k-1-q$ colors under c. By the minimality of k, $|H \backslash U| \leq 3 \cdot 2^{k-1-q}$. Thus $|U| \geq 2^{k-q} + 1 = 2 \cdot 2^{k-1-q} + 1$. Note that $|U| \geq 5$ and $G[U]$ has no blue edges because E is an induced matching in H. Thus the edges of $G[U]$ are colored by at most $k-1-q$ colors under c. By Theorem 1.5(b), $G[U]$ contains a monochromatic, say red, C_5. We may further assume that C_5 has vertices u_1, u_2, u_3, u_4, u_5 in order. Since $G[U]$ has no rainbow triangle under c, and E is an induced matching, we see that $\{u_1, v_1\}$ is red-complete to $\{u_2, v_2\}$ in G. We then obtain a red C_7 with vertices $u_1, v_2, v_1, u_2, u_3, u_4, u_5$ in order, a contradiction. Thus E is not an induced matching in H.

Finally, let P be a longest blue path in H with vertices $b_1, b_2, \ldots, b_q$ in order. Since E is not an induced matching in H and $|V(E)| \geq 5$, we see that $q \geq 3$. Let b_4b_5 be a blue edge in $H \backslash \{b_1, b_2, b_3\}$ if $q = 3$, and let $b \in V(E) \backslash \{b_1, b_2, b_3, b_4\}$ if $q \geq 4$. This is possible because $|V(E)| \geq 5$ and E is not an induced star in H. We then obtain a blue C_7 with vertices $u, b_1, b_2, b_3, v, b_4, b_5$ in order when $q = 3$; and vertices $u, b_1, b_2, b_3, b_4, v, b$ in order when $q \geq 4$, a contradiction. ■

Let $x_1, x_2, \ldots, x_m \in V(G)$ be a maximum sequence of vertices chosen as follows: for each $j \in [m]$, all edges between x_j and $V(G) \backslash \{x_1, \ldots, x_j\}$ are colored the same color under c. Let $X := \{x_1, x_2, \ldots, x_m\}$. Notice that X is possibly empty. For each $x_j \in X$, let $c(x_j)$ be the unique color on the edges between x_j and $V(G) \backslash \{x_1, \ldots, x_j\}$.

Claim 2. *$c(x_i) \neq c(x_j)$ for all $i, j \in [m]$ with $i \neq j$.*

Proof. Suppose that $c(x_i) = c(x_j)$ for some $i, j \in [m]$. We may assume that the color $c(x_i)$ is blue, $i < j$, and x_j is the first vertex in the sequence $x_1, \ldots, x_m$ such that $c(x_j) = c(x_i)$ for some $i \in [m]$ with $i < j$. By the pigeonhole principle, $j \leq k+1$. Let $A := \{x_1, x_2, \ldots, x_j\}$. By Claim 1, $|G \backslash A| \leq 3 \cdot 2^{k-1} + 2^k$, contrary to the fact that $|G \backslash A| = 3 \cdot 2^k + 1 - |A| \geq 3 \cdot 2^k + 1 - k - 1 > 3 \cdot 2^{k-1} + 2^k$. ■

By Claim 2, $|X| \leq k$. Let $G^* := G \backslash X$. Then G^* has no monochromatic copy of C_7 under c. Consider a Gallai-partition of G^*, as given in Theorem 1.1, with parts $A_1, A_2, \ldots, A_p$ such that $p \geq 2$ is as small as possible. We may assume that $1 \leq |A_1| \leq \cdots \leq |A_s| < 3 \leq |A_{s+1}| \leq \cdots \leq |A_p|$, where $0 \leq s \leq p$. Let $\mathcal{R}$ be the reduced graph of G^* with vertices $a_1, a_2, \ldots, a_p$. By Theorem 1.1, we may assume that the edges of $\mathcal{R}$ are colored red and blue. Notice that any monochromatic C_7 in $\mathcal{R}$ would yield a monochromatic C_7 in G^*. Thus $\mathcal{R}$ has no red or blue C_7. Since $R_2(C_7) = 13$, we see that $p \leq 12$. Then $|A_p| > 4$ because $|G| \geq 3 \cdot 2^4 + 1$. Thus $p - s \geq 1$.

Claim 3. *For any two disjoint sets $Y, Z \subseteq V(G)$ with $|Y|, |Z| \geq 3$, if all edges between Y and Z are colored the same color, say blue, then no vertex $x \in V(G) \backslash (Y \cup Z)$ is blue-complete to $Y \cup Z$ in G. Moreover, if $|Y| \geq 4$, then $G[Y]$ has no blue edges.*

Proof. Let $y_1, y_2, y_3 \in Y$ and $z_1, z_2, z_3 \in Z$. Suppose that there exists a vertex $x \in V(G) \backslash (Y \cup Z)$ such that x is blue complete to $Y \cup Z$ in G. Then we obtain a blue C_7 with vertices $x, y_1, z_2, y_2, z_3, y_3, z_1$ in order, a contradiction. Next suppose that $|Y| \geq 4$ and $G[Y]$ has a blue edge, say y_1y_2. We obtain a blue C_7 with vertices $z_1, y_1, y_2, z_2, y_3, z_3, y_4$ in order, where $y_4 \in Y \backslash \{y_1, y_2, y_3\}$, a contradiction. ■

Claim 4. *$p \geq s + 3$.*

Proof. Suppose that $p \leq s + 2$. Since $p - s \geq 1$, we see that $p = s + 1$ or $p = s + 2$. Let $A_b := \{a_i \in \{a_1, \ldots, a_s\} : a_ia_p$ is colored blue in $\mathcal{R}\}$ and $A_r := \{a_j \in \{a_1, \ldots, a_s\} : a_ja_p$ is colored red in $\mathcal{R}\}$. Let $C := \bigcup_{a_i \in A_b} A_i$ and $D := \bigcup_{a_j \in A_r} A_j$.

Suppose first that $p = s + 1$. Then $s \leq 11$ and so $|C \cup D| \leq 22$. If $|C| \geq 3$ or $|D| \geq 3$, say the former, by Claim 3 applied to C and A_p, we see that $G[A_p]$ has no blue edges, and no vertex $x \in X$ has $c(x)$ being the blue color. By the choice of k, $|A_p \cup X| \leq 3 \cdot 2^{k-1}$, contrary to the fact that $|A_p \cup X| = 3 \cdot 2^k + 1 - |C \cup D| \geq 3 \cdot 2^k + 1 - 22 > 3 \cdot 2^{k-1}$. Thus $|C| \leq 2$ and $|D| \leq 2$. By the choice of X, either $|C| = 2$ or $|D| = 2$, say the former. By Claim 1, $|A_p| \leq 3 \cdot 2^{k-1} + 2^k$, contrary to the fact that $|A_p| = 3 \cdot 2^k + 1 - |C \cup D| - |X| \geq 3 \cdot 2^k + 1 - 4 - k > 3 \cdot 2^{k-1} + 2^k$.

Next suppose that $p = s + 2$. We may assume that $a_{p-1}a_p$ is colored blue in $\mathcal{R}$. Then $a_{p-1} \in A_b$ and so $A_{p-1} \subseteq C$. Thus $|C| \geq |A_{p-1}| \geq 3$. By Claim 3, no vertex $x \in X$ has $c(x)$ being the blue color, and $G[A_p]$ has no blue edges. We claim that $G[D]$ has blue edges. Suppose that $G[D]$ has no blue edges. Then $G[A_p \cup D \cup X]$ has no blue edges. By the choice of k, $|A_p \cup D \cup X| \leq 3 \cdot 2^{k-1}$, and so $|C| \geq 3 \cdot 2^{k-1} + 1$. By Claim 3, $G[C]$ has no blue edges. By the choice of k, $G[C]$ must have a monochromatic C_7, a contradiction. Thus $G[D]$ has blue edges, as claimed.

Since $G[D]$ has blue edges, we have $|D| \geq 2$. Suppose that $|D| \geq 4$. By Claim 3, $G[A_p]$ and $G[D]$ have no red edges, and no vertex $x \in X$ has $c(x)$ being the red color. In particular, $\mathcal{R}[A_r]$ has no red edges and so all the edges in $\mathcal{R}[A_r]$ are colored blue. Since $\mathcal{R}$ has no blue C_7, we see that $|A_r| \leq 6$ and so $|D| \leq 12$. Since $G[A_p \cup X]$ has neither red nor blue edges, by the choice of k, $|A_p \cup X| \leq 3 \cdot 2^{k-2}$. Hence $|C| \geq 3 \cdot 2^k + 1 - 12 - 3 \cdot 2^{k-2} \geq 3 \cdot 2^{k-1} + 1$. By Claim 3, $G[C]$ has no blue edges. By the choice of k, $G[C]$ contains a monochromatic C_7, a contradiction. This proves that $2 \leq |D| \leq 3$.

Note that $G[A_p]$ has no blue edges. By Claim 1 applied to D and $G[A_p]$, $|A_p| \leq 3 \cdot 2^{k-2} + 2^{k-1}$. Thus $|C \cup X| \geq 3 \cdot 2^k + 1 - 3 - 3 \cdot 2^{k-2} - 2^{k-1} > 3 \cdot 2^{k-1} + 1$. By Claim 3, $G[C \cup X]$ has no blue edges. By the choice of k, $G[C \cup X]$ contains a monochromatic C_7, a contradiction. ■

By Claim 4, $p \geq s + 3$, and so $3 \leq |A_{p-2}| \leq |A_{p-1}| \leq |A_p|$. By Claim 3, $\mathcal{R}[\{a_{p-2}, a_{p-1}, a_p\}]$ is not a monochromatic triangle. Let B_1, B_2, B_3 be a permutation of A_{p-2}, A_{p-1}, A_p such that B_2 is, say, blue-complete, to $B_1 \cup B_3$ in G. Then B_1 is red-complete to B_3 in G. We may assume that $|B_1| \geq |B_3|$. By Claim 3, no vertex $x \in X$ has $c(x)$ being red or blue. Let $B := V(G) \backslash (B_1 \cup B_2 \cup B_3 \cup X)$. Then by Claim 3, no vertex in B is red-complete to $B_1 \cup B_3$ in G, and no vertex in B is blue-complete to $B_1 \cup B_2$ or $B_2 \cup B_3$ in G. This implies that B must be red-complete to B_2 in G. We next show that $G[B]$ has no blue edges. Suppose that $G[B]$ has a blue edge, say, uv. Let $B_1^* := \{b \in B : b$ is blue-complete to B_1 only$\}$, $B_2^* := \{b \in B : b$ is blue-complete to $B_1 \cup B_3\}$, and $B_3^* := \{b \in B : b$ is blue-complete to B_3 only$\}$. Notice that B_1^*, B_2^*, B_3^* are possibly empty, $B = B_1^* \cup B_2^* \cup B_3^*$, B_1^* is red-complete to B_3, and B_3^* is red-complete to B_1. Let $b_1, b_2 \in B_1, b_3, b_4 \in B_2$, and $b_5, b_6 \in B_3$. If uv is an edge in $G[B_1^* \cup B_2^*]$, then we obtain a blue C_7 with vertices $b_1, u, v, b_2, b_3, b_5, b_4$ in order, a contradiction. Similarly, uv is not an edge in $G[B_2^* \cup B_3^*]$. Thus uv must be an edge in $G[B_1^* \cup B_3^*]$ with one end in B_1^* and the other in B_3^*. We may assume that $u \in B_1^*$ and $v \in B_3^*$. Then we obtain a blue C_7 with vertices $b_1, u, v, b_5, b_3, b_2, b_4$ in order, a contradiction. This proves that $G[B]$ has no blue edges.

Since $G[B]$ has no blue edges and B is red-complete to B_2, we see that $G[B_2 \cup B \cup X]$ has no blue edges. By the choice of k, $|B_2 \cup B \cup X| \leq 3 \cdot 2^{k-1}$. Then $|B_1| > 4$. Since $|B_3| \geq 3$, by Claim 3, $G[B_1]$ has neither red nor blue edges. By the choice of k, $|B_1| \leq 3 \cdot 2^{k-2}$ and so $|B_3| \leq |B_1| \leq 3 \cdot 2^{k-2}$. But then $|B_2 \cup B \cup X| \geq 3 \cdot 2^{k-1} + 1$, a contradiction.

This completes the proof of Theorem 2.1. ■

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