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Note

Erdős–Lovász Tihany Conjecture for graphs with forbidden holes[☆]

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ABSTRACT

A *hole* in a graph is an induced cycle of length at least 4. Let $s \geq 2$ and $t \geq 2$ be integers. A graph G is (s, t)-*splittable* if $V(G)$ can be partitioned into two sets S and T such that $\chi(G[S]) \geq s$ and $\chi(G[T]) \geq t$. The well-known Erdős–Lovász Tihany Conjecture from 1968 states that every graph G with $\omega(G) < \chi(G) = s + t - 1$ is (s, t)-splittable. This conjecture is hard, and few related results are known. However, it has been verified to be true for line graphs, quasi-line graphs, and graphs with independence number 2. In this paper, we establish more evidence for the Erdős–Lovász Tihany Conjecture by showing that every graph G with $\alpha(G) \geq 3$, $\omega(G) < \chi(G) = s + t - 1$, and no hole of length between 4 and $2\alpha(G)-1$ is (s, t)-splittable, where $\alpha(G)$ denotes the independence number of a graph G.

1. Introduction

All graphs considered in this paper are finite and without loops or multiple edges. For a graph G, we use $V(G)$ to denote the vertex set, $E(G)$ the edge set, $|G|$ the number of vertices, $\Delta(G)$ the maximum degree, $\alpha(G)$ the independence number, $\omega(G)$ the clique number, $\chi(G)$ the chromatic number, and $\overline{G}$ the complement of G. For a vertex $x \in V(G)$, we will use $N(x)$ to denote the set of vertices in G which are adjacent to x. We define $N[x] = N(x) \cup \{x\}$ and $d(x) = |N(x)|$. Given vertex sets $A, B \subseteq V(G)$, we say that A is *complete to* (resp. *anti-complete to*) B if for every $a \in A$ and every $b \in B$, $ab \in E(G)$ (resp. $ab \notin E(G)$). The subgraph of G induced by A, denoted $G[A]$, is the graph with vertex set A and edge set $\{xy \in E(G) : x, y \in A\}$. We denote by $B\backslash A$ the set $B - A$, and $G\backslash A$ the subgraph of G induced on $V(G)\backslash A$, respectively. If $A = \{a\}$, we simply write $B\backslash a$ and $G\backslash a$, respectively. A graph H is an *induced subgraph* of a graph G if $V(H) \subseteq V(G)$ and $H = G[V(H)]$. A graph G is *claw-free* if G does not contain $K_{1,3}$ as an induced subgraph. Given two isomorphic graphs G and H, we may (with a slight but common abuse of notation) write $G = H$. A cycle with $t \geq 3$ vertices is denoted by C_t. We use the convention "A :=" to mean that A is defined to be the right-hand side of the relation.

Let $s \geq 2$ and $t \geq 2$ be integers. A graph G is (s, t)-*splittable* if $V(G)$ can be partitioned into two sets S and T such that $\chi(G[S]) \geq s$ and $\chi(G[T]) \geq t$. In 1966, the following conjecture of Lovász was published by Erdős [10] and is known as the Erdős–Lovász Tihany Conjecture.

Conjecture 1.1. *Let G be a graph with $\omega(G) < \chi(G) = s+t-1$, where $s \geq 2$ and $t \geq 2$ are integers. Then G is (s, t)-splittable.*

To date, Conjecture 1.1 has been shown to be true only for values of $(s, t) \in \{(2, 2), (2, 3), (2, 4), (3, 3), (3, 4), (3, 5)\}$. The case $(2, 2)$ is trivial. The case $(3, 3)$ was shown by Brown and Jung in 1969 [3]. Mozhan [15] and Stiebitz [19] each

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independently showed the case $(2, 4)$ in 1987. The cases $(2, 3)$, $(3, 4)$, and $(3, 5)$ were also settled by Stiebitz in 1987 [20]. Recent work on the Erdős–Lovász Tihany Conjecture has focused on proving the conjecture for certain classes of graphs. A graph G is a *quasi-line graph* if for every vertex $v \in V(G)$, the set of neighbors of v can be covered by two cliques, namely the vertex set of the neighborhood of v can be partitioned into two cliques. By definition, quasi-line graphs are claw-free. Recently, quasi-line graphs attracted more attention (see [4,5,9]). In particular, Chudnovsky and Seymour [9] gave a constructive characterization of quasi-line graphs. Kostochka and Stiebitz [14] showed that Conjecture 1.1 holds for line graphs. Balogh, Kostochka, Prince, and Stiebitz [2] then showed that Conjecture 1.1 holds for all quasi-line graphs, and all graphs G with $\alpha(G) = 2$.

Theorem 1.2 ([2]). *Let G be a graph with $\omega(G) < \chi(G) = s + t - 1$, where $s \geq 2$ and $t \geq 2$ are integers. If G is a quasi-line graph or $\alpha(G) = 2$, then G is (s, t)-splittable.*

More recently, Chudnovsky, Fradkin, and Plumettaz [6] proved the following slight weaking of Conjecture 1.1 for claw-free graphs, the proof of which is long and relies heavily on the structure theorem for claw-free graphs developed by Chudnovsky and Seymour [8].

Theorem 1.3. *Let G be a claw-free graph with $\chi(G) > \omega(G)$. Then there exists a clique K with $|V(K)| \leq 5$ such that $\chi(G \setminus V(K)) > \chi(G) - |V(K)|$.*

The most recent result related to the Erdős–Lovász Tihany Conjecture is due to Stiebitz [21], who showed that for integers $s, t \geq 2$, any graph G with $\omega(G) < \chi(G) = s + t - 1$ contains disjoint subgraphs G_1 and G_2 of G with either $\chi(G_1) \geq s$ and $\mathrm{col}(G_2) \geq t$, or $\mathrm{col}(G_1) \geq s$ and $\chi(G_2) \geq t$, where $\mathrm{col}(H)$ denotes the coloring number of a graph H.

If we restrict $s = 2$ in Conjecture 1.1, then the Erdős–Lovász Tihany Conjecture states that for any graph G with $\chi(G) > \omega(G) \geq 2$, there exists an edge $xy \in E(G)$ such that $\chi(G\backslash\{x, y\}) \geq \chi(G) - 1$. To prove this special case of Conjecture 1.1, suppose for a contradiction that no such an edge exists. Then $\chi(G\backslash\{x, y\}) = \chi(G) - 2$ for every edge $xy \in E(G)$. This motivates the definition of double-critical graphs. A connected graph G is *double-critical* if for every edge $xy \in E(G)$, $\chi(G\backslash\{x, y\}) = \chi(G) - 2$. A graph G is *t-chromatic* if $\chi(G) = t$. We are now ready to state the following conjecture, which is referred to as the *Double-Critical Graph Conjecture*, due to Erdős and Lovász [10].

Conjecture 1.4. *Let G be a double-critical, t-chromatic graph. Then $G = K_t$.*

Since Conjecture 1.4 is a special case of Conjecture 1.1, it has been settled in the affirmative for $t \leq 5$ [15,19], for line graphs [14], for quasi-line graphs, and for graphs with independence number two [2]. Representing a weakening of Conjecture 1.4, Kawarabayashi, Pedersen, and Toft [13] have shown that any double-critical, t-chromatic graph contains K_t as a minor for $t \in \{6, 7\}$. As a further weakening, Pedersen [16] showed that any double-critical, 8-chromatic graph contains K_8^- as a minor. Albar and Gonçalves [1] later proved that any double-critical, 8-chromatic graph contains K_8 as a minor. Their proof is computer-assisted. Rolek and the present author [18] gave a computer-free proof of the same result and further showed that any double-critical, t-chromatic graph contains K_9 as a minor for all $t \geq 9$. We note here that Theorem 1.3 does not completely settle Conjecture 1.4 for all claw-free graphs. Recently, Huang and Yu [12] proved that the only double-critical, 6-chromatic, claw-free graph is K_6. Rolek and the present author [17] further proved that the only double-critical, t-chromatic, claw-free graph is K_t for all $t \leq 8$.

In this paper, we establish more evidence for the Erdős–Lovász Tihany Conjecture. By Theorem 1.2, Erdő s–Lovász Tihany Conjecture holds for graphs G with $\alpha(G) = 2$ but remains unknown for graphs G with $\alpha(G) \geq 3$. Let $\mathcal{F}$ be a family of graphs. A graph is *$\mathcal{F}$-free* if it does not contain any $F \in \mathcal{F}$ as an induced subgraph. We prove the following main result.

Theorem 1.5. *Let G be a graph with $\alpha(G) \geq 3$ and $\omega(G) < \chi(G) = s + t - 1$, where $s \geq 2$ and $t \geq 2$ are integers. If G is $\{C_4, C_5, C_6, \ldots, C_{2\alpha(G)-1}\}$-free, then G is (s, t)-splittable.*

We prove Theorem 1.5 in Section 2. Our proof of Theorem 1.5 relies on Theorem 1.2 and the following well-known Strong Perfect Graph Theorem [7]. A *hole* in a graph G is an induced cycle of length at least 4. An *antihole* in G is an induced subgraph isomorphic to the complement of a hole. A graph G is *perfect* if $\chi(H) = \omega(H)$ for every induced subgraph H of G.

Theorem 1.6 ([7]). *A graph is perfect if and only if it has no odd hole and no odd antihole.*

We shall need the following corollary which was observed in [22].

Corollary 1.7 ([22]). *If G is $\{C_4, C_5, C_7, \ldots, C_{2\alpha(G)+1}\}$-free, then G is perfect.*

2. Proof of Theorem 1.5

The main idea in the proof of Theorem 1.5 is similar to that used in the proof of a result of Thomas and the present author (see Theorem 2.3 in [22]), which states that Hadwiger's Conjecture [11] is true for $\{C_4, C_5, C_6, \ldots, C_{2\alpha(G)-1}\}$-free graphs G with $\alpha(G) \geq 3$. We will show that any minimal counterexample to Theorem 1.5 is a quasi-line graph. First, we establish Lemma 2.1, noting that it is deduced from the proof of Theorem 2.3 given in [22]. Our hope is that if a conjecture has been proven true for quasi-line graphs, then Lemma 2.1 might be used to demonstrate that such a conjecture also holds for $\{C_4, C_5, C_6, \ldots, C_{2\alpha(G)-1}\}$-free graphs G with $\alpha(G) \geq 3$.

Lemma 2.1. *Let G be a $\{C_4, C_5, C_6, \ldots, C_{2\alpha(G)-1}\}$-free graph with $\alpha(G) \geq 3$ and $\Delta(G) \leq |G| - 2$. If G contains an induced cycle of length $2\alpha(G) + 1$, then G is a quasi-line graph.*

Proof. Let G be given as in the statement. Let C be an induced cycle of length $2\alpha + 1$ in G with vertices $u_0, u_1, \ldots, u_{2\alpha}$ in order, where $\alpha := \alpha(G)$. By assumption, $\Delta(G) \leq |G| - 2$. Let $W := V(G)\backslash V(C)$. Assume first that $W = \emptyset$. Then $G = C$ is a quasi-line graph, as desired. So we may assume that $W \neq \emptyset$. We next prove several claims.

(1) For every vertex $x \in W$, either x is complete to $V(C)$, or x is adjacent to exactly three consecutive vertices on C, or x is adjacent to exactly four consecutive vertices on C.

Proof. The statement is trivially true if W is complete to $V(C)$. Let $x \in W$ be a vertex such that x is not complete to $V(C)$. Since $\alpha(G) = \alpha$ and $|C| = 2\alpha+1$, we see that x is adjacent to at least one vertex on C. We may assume that $xu_0 \notin E(G)$ but $xu_1 \in E(G)$. Then x is not adjacent to $u_{2\alpha}, u_{2\alpha-1}, \ldots, u_5$ because G is $\{C_4, C_5, \ldots, C_{2\alpha-1}\}$-free. If $xu_4 \in E(G)$, then x must be adjacent to u_2, u_3 because G is $\{C_4, C_5\}$-free. If $xu_4 \notin E(G)$, then again x must be adjacent to u_2, u_3 because $\alpha(G) = \alpha$. Thus x is adjacent to either u_1, u_2, u_3 or u_1, u_2, u_3, u_4 on C, as desired. ■

(2) No vertex in W is complete to $V(C)$.

Proof. Suppose that there exists a vertex $x \in W$ such that x is complete to $V(C)$. Since $\Delta(G) \leq |G| - 2$, there must exist a vertex $y \in W$ such that $xy \notin E(G)$. By (1), we may assume that y is adjacent to u_0, u_1, u_2. But then $G[\{x, u_0, y, u_2\}]$ is an induced C_4 in G, a contradiction. ■

Let $I := \{0, 1, \ldots, 2\alpha\}$. For each $i \in I$, let $A_i \subseteq W$ (possibly empty) denote the set of vertices in G adjacent to precisely u_i, u_{i+1}, u_{i+2} on C, and let $B_i \subseteq W$ (possibly empty) denote the set of vertices in G adjacent to precisely $u_i, u_{i+1}, u_{i+2}, u_{i+3}$ on C, where all arithmetic on indices here and henceforth is done modulo $2\alpha + 1$. By (1) and (2), $\{V(C), A_0, A_1, \ldots, A_{2\alpha}, B_0, B_1, \ldots, B_{2\alpha}\}$ partitions $V(G)$. Since $\alpha(G) = \alpha$ and G is $\{C_4, C_5, \ldots, C_{2\alpha-1}\}$-free, we see that

(3) For each $i \in I$, both $G[A_i]$ and $G[B_i]$ are cliques; and A_i is complete to $A_{i-1} \cup A_{i+1}$; and A_i is anti-complete to each A_j for all $j \in I\backslash\{i-2, i-1, i, i+1, i+2\}$; and

(4) For each $i \in I$, B_i is complete to $B_{i-1} \cup A_i \cup A_{i+1} \cup B_{i+1}$ and anti-complete to each B_j for all $j \in I\backslash\{i-1, i, i+1\}$.

We next show that

(5) For each $i \in I$, if $B_i \neq \emptyset$, then $B_j = \emptyset$ for all $j \in I\backslash\{i-2, i-1, i, i+1, i+2\}$. Moreover, B_i is anti-complete to A_j for all $i \in I$ and $j \in I\backslash\{i-1, i, i+1, i+2\}$.

Proof. Suppose there exist some $i \in I$ and $j \in I\backslash\{i-2, i-1, i, i+1, i+2\}$ such that $B_i \neq \emptyset$ and $B_j \neq \emptyset$. We may assume that $j > i+2$. Let $a \in B_i$ and $b \in B_j$. By (4), B_i is anti-complete to each B_j and so $ab \notin E(G)$. But then $G[\{u_i, a, u_{i+3}, \ldots, u_j, b, u_{j+3}, \ldots, u_{i-1}\}]$ is an induced $C_{2\alpha-1}$ in G, a contradiction. Similarly, suppose there exist some $i \in I$ and $j \in I\backslash\{i-1, i, i+1, i+2\}$ such that B_i is not anti-complete to A_j. We may assume that $i+2 < j$. Let $b \in B_i$ and $a \in A_j$ be such that $ba \in E(G)$. Then $G[\{u_i, a, b, u_{j+2}, \ldots, u_{i-1}\}]$ is an induced cycle of length between 4 and $2\alpha - 1$ in G, a contradiction. ■

(6) For each $i \in I$, every vertex in A_i is either anti-complete to A_{i+2} or anti-complete to A_{i-2}; and every vertex in B_i is either complete to A_{i-1} or complete to A_{i+2}.

Proof. Suppose first that there exists a vertex $x \in A_i$ such that x is adjacent to a vertex $y \in A_{i-2}$ and a vertex $z \in A_{i+2}$. But then $G[\{x, y, z\} \cup (V(C)\backslash\{u_{i-1}, u_i, u_{i+1}, u_{i+2}, u_{i+3}\})]$ is an induced $C_{2\alpha-1}$ in G, a contradiction. Suppose next that there exists a vertex $b \in B_2$ such that b is not adjacent to a vertex $a_1 \in A_1$ and a vertex $a_4 \in A_4$. By (3), A_1 is anti-complete to A_4. Thus G contains an independent set $\{b, a_1, a_4, u_0\}$ of size four when $\alpha = 3$ or an independent set $\{b, a_1, a_4, u_0, u_7, u_9, \ldots, u_{2\alpha-1}\}$ of size $\alpha + 1$ when $\alpha \geq 4$, a contradiction. ■

(7) There exists an $i \in I$ such that $B_j = \emptyset$ for all $j \in I\backslash\{i, i+1, i+2\}$.

Proof. The statement is trivially true if $B_j = \emptyset$ for all $j \in I$. So we may assume that $B_j \neq \emptyset$ for some $j \in I$, say $B_2 \neq \emptyset$. Then by (5), $B_j = \emptyset$ for all $j \in \{5, 6, \ldots, 2\alpha\}$. By (5) again, either $B_0 \neq \emptyset$ or $B_4 \neq \emptyset$ but not both. By symmetry, we may assume that $B_4 = \emptyset$. Similarly, either $B_0 \neq \emptyset$ or $B_3 \neq \emptyset$ but not both. Thus either $B_j = \emptyset$ for all $j \in I\backslash\{0, 1, 2\}$ or $B_j = \emptyset$ for all $j \in I\backslash\{1, 2, 3\}$. ■

By (7), we may assume that $B_j = \emptyset$ for all $j \in I\backslash\{1, 2, 3\}$. For each $i \in I$ with $A_i \neq \emptyset$, let $A_i^1 := \{a \in A_i :$ a has a neighbor in $A_{i-2}\}$, $A_i^3 := \{a \in A_i : a$ has a neighbor in $A_{i+2}\}$, and $A_i^2 := A_i\backslash(A_i^1 \cup A_i^3)$. Then A_i^2 is anti-complete to $A_{i-2} \cup A_{i+2}$. By (6), A_i^1 is anti-complete to A_{i+2} and A_i^3 is anti-complete to A_{i-2}. Clearly, $\{A_i^1, A_i^2, A_i^3\}$ partitions A_i. Next, for each $j \in \{1, 2, 3\}$, by (6), every vertex in B_j is either complete to A_{j-1} or complete to A_{j+2}. Let $B_j^1 := \{b \in B_j :$ b is complete to $A_{j-1}\}$ and $B_j^2 := \{b \in B_j : b$ is complete to $A_{j+2}\}$. Clearly, B_j^1 and B_j^2 are not necessarily disjoint. It is worth noting that B_j^1 and B_j^2 are not symmetrical because B_j^1 is complete to A_{j-1} and B_j^2 is complete to A_{j+2}.

(8) For each $j \in \{1, 2, 3\}$, B_j is anti-complete to $A_{j-1}^1 \cup A_{j+2}^3$.

Proof. Suppose there exist a vertex $b \in B_j$ and a vertex $a \in A_{j-1}^1 \cup A_{j+2}^3$ such that $ba \in E(G)$. By the definition of A_{j-1}^1 and A_{j+2}^3, we see that a has a neighbor, say c, in A_{j-3} if $a \in A_{j-1}^1$, and in A_{j+4} if $a \in A_{j+2}^3$. But then, either $G[\{b, a, c\} \cup (V(C)\backslash\{u_{j-2}, u_{j-1}, u_j, u_{j+1}, u_{j+2}\})]$ is an induced $C_{2\alpha-1}$ in G when $a \in A_{j-1}^1$, or $G[\{b, a, c\} \cup (V(C)\backslash\{u_{j+1}, u_{j+2}, u_{j+3}, u_{j+4}, u_{j+5}\})]$ is an induced $C_{2\alpha-1}$ in G when $a \in A_{j+2}^3$. In either case, we obtain a contradiction. ■

By the above claims, one can verify that G is a quasi-line graph, using the same argument of Claim 15 given in the proof of Theorem 2.3 in [22]. This completes the proof of Lemma 2.1. ■

We are now ready to prove Theorem 1.5.

Proof of Theorem 1.5. The statement is trivially true when $s = t = 2$. So we may assume that $t \geq 3$. Then $\chi(G) = s + t - 1 \geq 4$. Suppose for a contradiction that G is not (s, t)-splittable. Let G be a counterexample with $\chi(G)$ minimum. By Theorem 1.6 and the assumption that $\chi(G) > \omega(G)$, G is not perfect. Since G is $\{C_4, C_5, C_7, \ldots, C_{2\alpha(G)-1}\}$-free, by Corollary 1.7, we see that G must contain an induced cycle of length $2\alpha(G) + 1$. We next claim that $\Delta(G) \leq |G| - 2$. Suppose there exists a vertex x in G with $d(x) = |G| - 1$. Then $\chi(G\backslash x) = \chi(G) - 1 = s + (t - 1) - 1$ because $\chi(G) = s + t - 1$. Since $t \geq 3$, by minimality of $\chi(G)$, $G\backslash x$ is $(s, t - 1)$-splittable. Let $\{S, T'\}$ be a partition of $V(G\backslash x)$ such that $\chi(G[S]) \geq s$ and $\chi(G[T']) \geq t - 1$. Let $T := T' \cup \{x\}$. Then $\{S, T\}$ is a partition of $V(G)$ with $\chi(G[S]) \geq s$ and $\chi(G[T]) = \chi(G[T']) + 1 \geq t$. Thus G is (s, t)-splittable, a contradiction. This proves that $\Delta(G) \leq |G| - 2$, as claimed. By Lemma 2.1, G is a quasi-line graph. By Theorem 1.2, G is (s, t)-splittable, a contradiction. This completes the proof of Theorem 1.5. ■

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Conflict of interest

None.

Declaration of competing interest

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