ED1 Problems

Section 1.

1. See A. A. Michelson, Am. J. Sci. 122, 120-129 (1881). What fraction of the wavelength of yellow light was he trying to measure and how would this show whether or not there was ether? What was the upper bound on interference-fringe shift determined from his measurements?
2. Summarize Ch. Eisele, A. Yu. Nevsky, and S. Schiller, "Laboratory Test of the Isotropy of Light Propagation at the 10^{-17} Level, Phys. Rev. Lett. 103, 090401 (2009) in one page.
3. Classical principle of relativity and free particles. Let K be an inertial reference frame. (a) Suppose a frame K' moves with constant velocity relative to K. Consider a free particle and use Gallileo's velocity addition rule to show that K' is also an inertial reference frame. (b) Conversely, if K' is an inertial frame, show that it moves at constant velocity with respect to K.
4. Classical principle of relativity and collisions. In inertial frame K, particle A (mass m_A and velocity $\mathbf{v_A}$) hits particle B (m_B, $\mathbf{v_B}$). Some mass dm rubs off A onto B, leaving particles C (m_C, $\mathbf{v_C}$) and D (m_D, $\mathbf{v_D}$). Assume momentum and energy conservation hold in frame K. Frame K' moves relative to K at velocity $\mathbf{V}$. (a) Use Gallileo's velocity addition rule to show that momentum is also conserved in K'. What must you assume about the total mass? (b) Suppose the collision is elastic in K. Show that it is also elastic in K'.
5. Show that the apparent speed of a star across the sky can exceed c. With what angle to the line of sight should the star move to give the maximum apparent speed?

Section 2.

1. Draw a space-time diagram representing a game of catch for two people at rest 10 m apart, i.e. sketch the world line of the ball. Indicate also the light cone on the graph (need not be to scale).
2. Event A happens at (ct,x,y,z)=(15,5,3,0); event B at (5,10,8,0); both in the K system.
   a. What is the interval between the two events?
   b. Is there an inertial system K' in which they occur simultaneously? If so what is its velocity vector relative to K?
   c. Is there an inertial system K' in which they occur at the same spatial position? If so, what is its velocity vector relative to K?
3. Two observers K and K' observe two events, A and B. The observers have a constant relative speed of 0.8 c. In units such that the speed of light is 1, observer K obtained the following coordinates: Event A, (x,y,z,t)=(3,3,3,3); Event B, (5,3,1,5). What is the space-time interval between these two events as measured by observer K'? Is it space-like or time-like? Can Event A have caused Event B?
4. Find a translation of the original work by H. Minkowski that introduced four-dimensional geometry. Hint: Look in <u>Principle of Relativity: A Collection of Original Memoirs on the Special and General Theory of Relativity</u>, by H A Lorentz, Albert Einstein, H Minkowski, Herman Weyl for the citation. Are there

any obvious differences between Minkowski's original presentation and the distilled version that appears in Landau & Lifshitz?

Section 3.

1. The mean lifetime of muons = 2 μs in their rest frame. Muons are generated in the upper atmosphere as cosmic ray secondaries. (a) Calculate the mean distance traveled by muons with speed 0.99 c assuming classical physics (c=infinity). (b) Calculate the mean distance using special relativity. (c) What percentage of muons produced at an altitude of 10 km reach the ground assuming they travel straight down at 0.99 c.
2. In a laboratory experiment a muon is observed to travel 800 m before disintegrating. A student looks up the muon lifetime (2 μs) and concludes that its speed was 4×10^8 m/s, which exceeds c. Identify the error, and find the actual speed of the muon.
3. A rocket leaves earth at speed 3c/5. When a clock on the rocket says 1 hour has elapsed, the rocket ship sends a light signal back to earth. According to earth clocks, when was the signal sent? According to earth clocks, how long after the rocket left did the signal arrive back on earth? According to the rocket observer, how long after the rocket left did the signal arrive back on earth?
4. A muon is traveling through the lab at 3c/5. How long does it last if its lifetime in its own rest frame is 2 μs?
5. Tau leptons are observed to have an average half-life of Δt_1 in the frame S_1 in which the leptons are at rest. In an inertial frame S_2, which is moving at a speed v_{12} relative to S_1, the leptons are observed to have an average half-life of Δt_2. In another inertial reference frame S_3, which is moving at a speed v_{13} relative to S_1 and v_{23} relative to S_2, the leptons have an observed half-life of Δt_3. Which of the following is a correct relationship among two of the half-lives, Δt_1, Δt_2, and Δt_3? A) $\Delta t_2=\Delta t_1$Sqrt$[1-v_{12}^2/c^2]$; B) $\Delta t_1=\Delta t_3$Sqrt$[1-v_{13}^2/c^2]$; C) $\Delta t_2=\Delta t_3$Sqrt$[1-v_{23}^2/c^2]$; D) $\Delta t_3=\Delta t_2$Sqrt$[1-v_{23}^2/c^2]$; E) $\Delta t_1=\Delta t_2$Sqrt$[1-v_{23}^2/c^2]$;
6. GPS satellites orbit the earth with a speed of 14000 km/hr relative to fixed positions on earth. Is this a relativistic speed? By how much do clocks on the satellites lag behind clocks on earth per day? If the timing of signals from the satellites received by an earth receiver must have 20 ns precision to determine position within 5 meters, can special relativity be ignored? How long before the difference in time measurements would exceed the required precision?

Section 4.

1. Inertial system K' moves at constant velocity **V** = V Cos[ϕ] **x**^ + V Sin[ϕ] **y**^ with respect to inertial system K. The axes of the K and K' systems are parallel. Their origins coincide at t = t' = 0. For convenience, use abbreviations $\beta = V/c$ and $\gamma = 1/\text{Sqrt}[1-\beta^2]$. Find the Lorentz transformation matrix Λ, i.e. find Λ such that (ct,x,y,z) = Λ(ct', x', y', z').
2. Show that the Galileo transformation does not leave the interval between events invariant? Show that the Lorentz transform leaves $(ct)^2-x^2$ unchanged.

3. Which of the following represents a Lorentz transformation, assuming c=1? (x',y',z',t')= A) (4x, y, z, t/4); B) (x - 3t/4, y, z, t); C) (5x/4 - 3t/4, y, z, 5t/4-3x/4); D) (5x/4 – 3t/4, y, z, 3t/4 – 5x/4), E) None of the above.
4. A bus of rest length 5 m passes through a garage of rest length 4 m. Due to Lorentz contraction, the bus is only 3 meters long in the garage's rest frame. What is the velocity of the bus in the garage's rest frame? What is the length of the garage in the bus's rest frame?
5. The coordinate systems K_1 and K_2 move along the X-axis of a reference coordinate frame K, with velocities v_1 and v_2 respectively, referred to K. The time measured in K for the hand of a clock in K_1 to go around once is t. What is the time interval t_2 measured in K_2 for the hand to go around, in terms of t, v_1, v_2, and c. (Hint: The time dilation formula approach is messy since the velocity of K_2 relative to K_1 is *not* simply equal to v_2-v_1. Use Lorentz transformation formulas instead.)
6. A bar lies along the X' axis and is stationary in the K' system. Show that if the positions of its ends are observed in K at instants which are simultaneous in K', its length deduced from these observations will be greater than its length in K' by a factor $(1-V^2/c^2)^{-1/2}$.
7. Sophie feels a pain at the same instant her brother, located 500 km away to the West, hits his thumb with a hammer. A pilot in an airplane traveling at 12c/13 to the East observes both events. Which event occurred first according to the pilot? How much earlier was it in seconds? Could hammer blow have caused Sophie's pain?

Section 5.

1. Show that the sum of two velocities, each smaller than c, is also smaller than c.
2. For V<<c, verify expression (5.3) for the transformation of the velocity vector and the equations preceding it.
3. Derive the expression (5.4) for the change in the direction of the velocity on transforming from one reference system to another.
4. Derive the formula for the aberration of light (5.7) and the formula that precedes it.
5. A π^0 meson (rest-mass energy 135 MeV) is moving with velocity 0.8 c in the z direction in the laboratory rest frame when it decays into two photons, γ_1 and γ_2. In the π^0rest frame, γ_1 is emitted forward and γ_2 is emitted backward relative to the π^0 direction of flight. What is the velocity of γ_2 in the laboratory rest frame?
6. In the frame K, at t = 1, a particle leaves the origin O and moves with constant velocity in the XY-plane having components $v_x = 5c/6$, $v_y = c/3$. What are the coordinates (x,y) of the particle at any later time t? If the velocity of K' relative to K is V = 3c/5, calculate the coordinates (x', y') of the particle at time t' in K' and deduce that the closest approach of the particle to O' occurs at time t' = 220/113. Origins of K and K' coincide at t = t' = 0.

7. Obtain the transformation equations for the acceleration **a** by differentiating the transformation equations for **v**.
8. A nucleus is moving with speed 3c/5. It emits a β-particle in a direction perpendicular to the line of motion of the nucleus as observed from the reference frame of the nucleus. The speed of the β-particle in this frame is 3c/4. Find the velocity and direction of motion of the β-particle as seen by a stationary observer in the lab frame.
9. A luminous disk of radius *a* has its center fixed at the point (x',0,0) of the K'-frame, which moves at speed V along the common X,X' axes relative to the K-frame. The plane of the disk is perpendicular to the X'-axis. It is observed from the origin of the K-frame, at the instant the origins of K and K' coincide, that the disk subtends an angle 2θ. If $a \ll x'$, show that tanθ = tanθ' Sqrt[(c-V)/(c+V)]. If the disk is moving away from the K observer, does the disk appear larger or smaller than to the K' observer? What if the disk is moving toward the K observer?
10. Atoms at rest emit photons isotropically. For an observer watching a beam of atoms moving at speed 0.9 c relative to the lab, does the emission appear isotropic to an observer in the lab? For photons emitted at angles 0,30,60,90,120,150,180 in the reference frame of the atoms, what angles of emission does the observer see? Describe qualitatively and draw the distribution of emission. Will atoms appear brighter as they approach or recede from the observer?
11. What is the percent error introduced when you use the Galileo, instead of the Einstein, velocity addition rule? What is the numerical error if v_x = 5 mi/hr and V = 60 mi/hr?
12. Suppose you could run at half the speed of light down the corridor of a train going three-quarters the speed of light. What would your speed be relative to the ground?
13. A bank robber's getaway car travels 3c/4 relative to the ground, and the police car pursuing him only c/2. So the policeman fires a bullet after the robber, and the bullet has a muzzle velocity relative to the gun of c/3. Does the bullet reach its target (a) according to Galileo? (b) according to Einstein?

Section 6. Landau problems 6.1 and 6.2 in book.

1. Prove that the symmetry (or antisymmetry) of a tensor is preserved by Lorentz transformation.
2. Use the $e_{\alpha\beta\gamma}$ tensor method to prove the product rule
   $\nabla \times (f\mathbf{A}) = f(\nabla \times \mathbf{A}) - \mathbf{A} \times (\nabla f)$, where f is a scalar function of coordinates and **A** is a vector function of coordinates.
3. Consider an integral over a surface in 3-space. Let the vectors d**r** = [dx, 0, 0] and d**r**' = [0, dy, 0] define an area element.
   a. Find the tensor $df_{\alpha\beta}$ that gives the projections of the area element on the $x_\alpha x_\beta$ planes.
   b. What is the projection of the area element on the x_1x_2 = xy plane? On the x_1x_3 plane?

c. Find the vector df_α that is dual to $df_{\alpha\beta}$. What is its magnitude and direction relative to the area element?

4. Consider a volume element determined by the vectors d**r**=dx **x**, d**r**'=dy **y**, d**r**"=dz **z**. Show that the determinant of 3rd rank formed from the components of these vectors gives the volume of the parallelepiped spanned by these vectors.
5. Show that the Lorentz transform leaves the 4-D volume element unchanged.
6. Show that $(1/2)\,(dS_i\, \partial A^{ik}/ \partial x^k - dS_k\, \partial A^{ik}/ \partial x^i) = dS_i\, \partial A^{ik}/ \partial x^k$, where A^{ik} is antisymmetric.
7. Show that $df^{ki}\, \partial A_i/ \partial x^k = (1/2)\, df^{ik}\, (\partial A_k/ \partial x^i - \partial A_i/ \partial x^k)$, which is analogous to the (surface element * Curl) expression in Stokes theorem.
8. Show that in 2D, the general orthogonal transformation as matrix A given by $\{\{\cos\theta, \sin\theta\}, \{-\sin\theta, \cos\theta\}\}$. Verify that det[A] = 1 and that the transpose of A equals its inverse. Let T_{ij} be a tensor in this space. Write down in full the transformation equations for all its components and deduce that T_{ii} is an invariant.
9. A_{ijk} is a tensor, all of whose components are zero, except for $A_{111} = A_{222} = 1$, $A_{212} = -2$. Calculate the components of the vector A_{iji}. A necessary condition for a transformation to be orthogonal is that its determinant = 1 and that its transpose equals its inverse. Show that the transformation $x'_1 = (1/7)\,(-3\, x_1 - 6\, x_2 - 2\, x_3)$, $x'_2 = (1/7)\,(-2\, x_1 + 3\, x_2 - 6\, x_3)$, $(1/7)\,(6\, x_1 - 2\, x_2 - 3\, x_3)$ has these properties. Calculate component A'_{123} in the x'-frame. If B_{ij} is a tensor whose components in the x' frame all vanish except $B'_{13} = 1$, calculate B_{12}.
10. Show that the components of the metric tensor are the same in all coordinate systems. Hint: See L&L Problem 1.
11. Covariant and contravariant vectors exist already in 3D, though there is usually no distinction except in oblique (non-orthogonal) coordinates or in curved space. A linear transformation of the coordinates such as a rotation transforms the components of a vector according to $A'_i = a_{ij}A_j$, where a_{ij} is the cosine of the angle between the x'_i and x_j axes. (a) **Show** using the chain rule that, for the differential displacement vector d**r,** $a_{ij} = \partial x'_i/\partial x_j$. Vectors that transform according to these coefficients are called contravariant. A scalar function has the same value in any coordinate system, $\phi = \phi(x,y,z) = \phi(x',y',z') = \phi'$. The gradient of ϕ is a vector. (b) **Show** that the components $\partial\phi'/\partial x'_i$ of this gradient transform according to differently-defined coefficients $b_{ij} = \partial x_j /\partial x'_i$. Vectors that transform according to these coefficients are called covariant. (c) **Show** that for a rotation in cartesian coordinates, $a_{ij} = b_{ij}$, i.e. there is no difference between covariant and contravariant transformations of cartesian coordinates in 3D.
12. Show that if the components of any 4-tensor of any rank vanish in one inertial frame K', then the components of that tensor are also zero in any other inertial reference frame.
13. Consider an isotropic 4-tensor A^{ik}. By considering first the effect of rotations by 90^o about the spatial coordinate axes, then a general Lorentz transformation, show that A^i_k, is proportional to δ^i_k.
14. The components of a 4-tensor A^{ij} are equal to the corresponding components of a 4-tensor B^{ij} in one particular inertial reference frame K. Are they still equal in any different inertial frame K'?

15. T^{iklm} is antisymmetric with respect to all pairs of indices. How many independent components does it have in 4 space?

Section 7.

1. A car travels along a 45 deg line in K at speed v = 2c/Sqrt[5].
   a. Find components of ordinary velocity vector **v**.
   b. Find components of the 4 velocity u^i.
   c. System K' moves in the X direction at speed V=Sqrt[2/5]c relative to K. Use the velocity transformation law to find the velocity of the car in K', i.e. find **v**'.
   d. Find u'^i for the car in K' using the Lorentz transform for 4 vectors. Check that the result agrees with Eq. (7.2).
2. Consider a particle in hyperbolic motion, x[t] = Sqrt[b^2 + $(ct)^2$], y=z=0. Find the proper time t' for the particle as a function of t, assuming the clocks are synchronized at t=t'=0. Find x and v (ordinary velocity) as functions of t'. Find the 4-velocity as a function of t.
3. From the transformation properties of the four-velocity of a particle, derive the transformation equations for the components of its ordinary three-dimensional velocity (5.1).
4. Find expressions for the 4-acceleration $w^i = (w^0, \mathbf{w})$ in terms of the 3-D velocity **v** and 3-D accelerations d**v**/dt. What are the dimensions of the 4-acceleration?

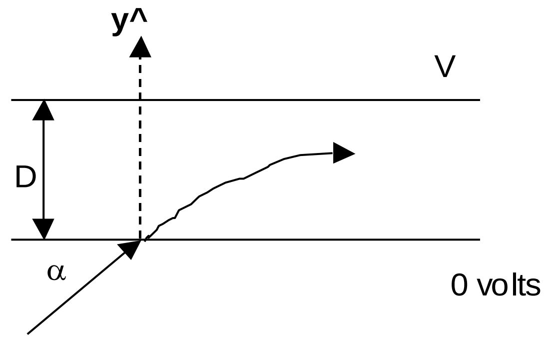
Section 8

1. From its relativistic Lagrangian, determine a free particle's equation of motion.

Section 9

1. Derive the relativistic Newton's 2nd law (9.2) in the case that the force acting on a particle is perpendicular to its velocity. Derive the relativistic Newton's 2nd law in the case that the force is parallel to the velocity (9.3). Show that each expression is a special case of $\mathbf{f} = (m/\sqrt{(1-v^2/c^2)})\, d\mathbf{v}/dt + (\mathbf{f}\bullet\mathbf{v})\, \mathbf{v}/c^2$.
2. If a particle with non-zero mass is ultra relativistic, show that its momentum is approximately its total energy divided by c.
3. Derive the expression (9.18) for the force 4-vector in terms of the usual 3-D force vector.
4. A monoenergetic beam consists of unstable particles with total energies 100 times their rest energy. If the particles have rest mass m, their momentum is most nearly A) mc; B) 10 mc; C) 70 mc; D) 100 mc; E) 10^4 mc. Explain.
5. A free electron (rest mass $m_e = 0.5$ MeV/c^2) has a total energy of 1.5 MeV. What is its momentum in units of MeV/c?
6. A positive kaon (K^+) has a rest mass of 494 MeV/c^2, whereas a proton has a rest mass of 938 MeV/c^2. If a kaon has a total energy that is equal to the proton rest energy, what is the speed of the kaon in units of c?
7. If a particle's kinetic energy is n times its rest energy, what is its speed?
8. A fast charge *e* enters the space between the plates of a parallel plate capacitor at an angle α (see figure) at time t = 0 and at point y = x = 0. (a) What is the electric field vector between the plates in terms of the given parameters? (b) Solve the

equations of motion for the relativistic momentum components for $t > 0$. (c) Find an expression for the kinetic energy ε_{kin} as a function of time and initial kinetic energy ε_0. (Hint: Use a relation between ε_{kin} and p). (d) Find an expression for the velocity components as a function of time. (Hint: use a relation between ε_{kin}, **p**, and **v**.) (e) Find the parametric expressions for x(t) and y(t). Plot the trajectory in the x,y plane.



9. Consider the momentum 4-vector $p^i = (\varepsilon/c, p)$. If **p**' = **p** = 0 in two inertial frames K' and K, show that $\varepsilon = 0$ in all inertial reference frames. From this result show that if momentum is conserved in two reference frames, energy is conserved in all reference frames.
10. If m is a particle's rest mass, **f** the ordinary 3D force vector, **v** the ordinary 3D velocity vector of the particle, and ε the relativistic kinetic energy, show that $\mathbf{v}\cdot\mathbf{f} = d\varepsilon/dt$. Hint: Use equations (9.13,17,18)

Section 15.

1. A disk of radius R rotates at angular velocity w about its symmetry axis. What is the ratio of the circumference to the diameter in terms of ω and R according to an observer at rest.

Section 16.

1. Find the scalar and vector potentials of a point charge e moving at constant velocity **V** relative to the lab reference frame. In its own reference frame, the potentials of the charge are $\phi' = e/r'$ and $\mathbf{A}'=0$. How would you solve for **E** and **H** in the lab frame?
2. Find the scalar and vector potentials of a point electric dipole **p** moving at constant velocity **V** relative to the lab. In its own reference frame, the potentials of the dipole are $\phi' = \mathbf{p}\bullet\mathbf{r}'/r'^3$ and $\mathbf{A}'=0$. Express answer in terms of the separation vector between the dipole and the field point in lab coordinates.
3. An ideal magnetic dipole moment **m** is located at the origin of an inertial system K' that moves with speed V in the X direction with respect to inertial system K. In K' the vector potential is $\mathbf{A}' = (\mathbf{m} \times \mathbf{r}')/r'^3$, where **r**' is the location of the field point in K', and the scalar potential $\phi'=0$. Find the scalar potential in the K system in terms of **R**, the instantaneous vector from **m** to the field point in the K system. Let **r** be the location of the field point in K.

Section 17.

1. Derive the vector identity grad(**a.b**)=(**a.**∇)**b** + (**b.**∇)**a** + **b** x curl **a** + **a** x curl **b** using the $e_{\alpha\beta\gamma}$ tensor method.
2. Show that $d\varepsilon_{kin}/dt$ = **v**.(d**p**/dt).
3. A laser pulse is represented by the four potential A^i = [0, 0, f(x-ct) 0], where the function f(x) approaches zero as x → ± ∞. The pulse hits an electron (-e, m, $\mathbf{r}_0$=0, $\mathbf{v}_0$=0). (a) What are the equations of motion for the three components of v (assuming v <<c)? (b) Show that $v_z(t) = 0$. (c) Show that $v_y(t)$ = (e/mc) f(x-ct), where x is the position of the electron at time t. (d) Show that $v_x (1-v_x/(2c)) \approx v_x = (e^2/2\, m^2\, c^3)\, [f(x-ct)]^2$. What is the sign of the net displacement along x? Sketch a possible trajectory of the particle, y vs x, assuming the function f is a smooth positive function, like a Gaussian or a Lorentzian. What happens at long times?
4. Suppose A^i = [0, 0, A_0 Sin[k x – ω t], 0**]**. Find **E** and **H**.
5. (a) Express the acceleration of a particle in terms of its velocity and the electric and magnetic field intensities. (b) What does this expression become to first order in (v/c)? (c) Suppose H=0, E=1 MV/m (which might be produced by a big van de Graaf generator), and the particle is an electron initially at rest. How long does it take for the electron to reach a speed of (4/5) c as predicted by the exact relativistic expression and as predicted by the non-relativistic expression?

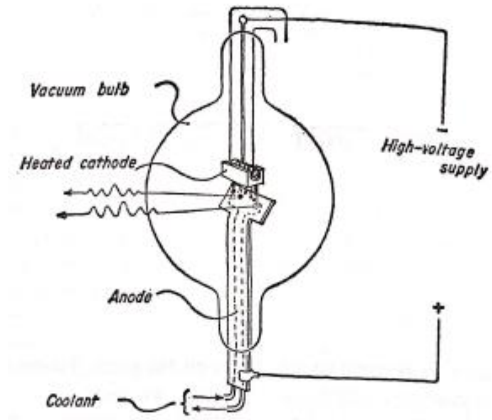
Section 18.

1. Show that the four potentials A^i = (e/r,**0**) and (0,-ct**r**/r^3) give the same fields. Starting from the Lagrangian in terms of these potentials, show that the equation of motion for a charge in fields given by these potentials is the same in each case.
2. The potentials for a point charge at the origin are usually given as A^i = (e/r, 0) in Gaussian units. Make a gauge transformation with f = cet/r. What are the new potentials? Anything surprising, given that this is an electrostatic situation? What are the fields **E** and **H** calculated from the new potentials, and how do they relate to the fields from the old potentials?

Section 19.

1. Consider the vector potential **A**[**r**] = **c** x **r** / 2, where **c** is a constant vector. What is the magnetic field?
2. The vector potential for a spinning sphere (radius a, angular velocity **ω**) with surface charge density σ is (in SI units) **A**(**r**) = (μ_0 σ a/3) (**ω** x **r**) for r<a and (μ_0 σ $a^4/(3\, r^3)$) (**ω** x **r**) for r>a. Calculate **B**(**r**) inside and out, sketch the field lines, and describe in words.
3. An electron starts from rest and is accelerated through a region of space with a 1 million volt potential drop. After passing through this region, what is the electron's velocity? Compare results of classical and relativistic calculations.
4. Suppose the 4-potential is A^i = (0,(A_0 a/r) Exp[-r^2/a^2] $\mathbf{e}_\phi$) in cylindrical coordinates. Calculate the fields **E** and **H**.
5. The 4-potential of a finite uniformly charged wire, length = 2 *l*, λ=charge per unit length, is A^i=(λ Log[*l* + Sqrt[r^2 + l^2]]/(-*1* + Sqrt[r^2+l^2])], **0**). Find the fields **E** and **H**.
6. Consider the image. What is it? (Hint: "1895"). What particles are emitted by the cathode? What is shown exiting the bulb? At typical high voltage in modern

versions of this device is 70 kV. What is the maximum velocity of the particles that hit the anode? Are they relativistic? What is v/c exactly? What is it classically? What is the % difference?



7. What particles are accelerated by the Cockroft-Walton machine from 1928 and used more recently as pre-accelerators? If the maximum accelerating voltage is 1.4 MV, what is the velocity these particles achieve? Is it relativistic? What is v/c exactly? What is it classically? What is the % difference? Explain how the Villard-cascade voltage multiplier consisting of a single transformer, diodes, and capacitors is used to achieve the necessary high voltage.

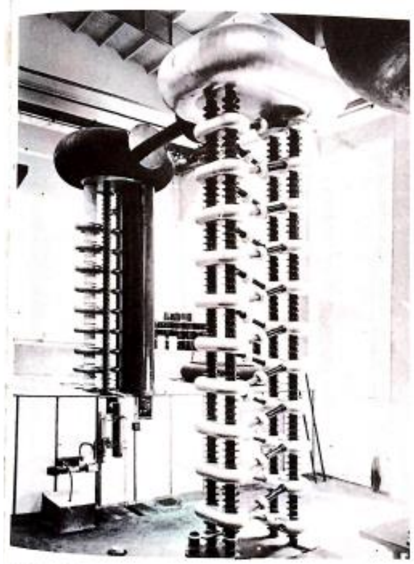


PLATE III. The 1.4-Mev Cockroft-Walton accelerator at Cavendish Laboratory, Cambridge, England. The white tube with rings at the left is the accelerating tube. At its top is the ion source in the high-voltage terminal. The black column to the right houses belt driving generator in terminal. The assemblage at the right consists of two condenser stacks with rectifier tubes. Experiments with accelerated ions are conducted in room below.

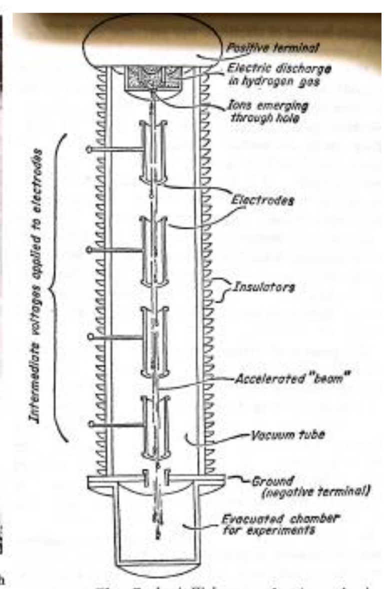
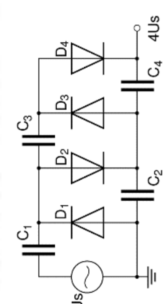
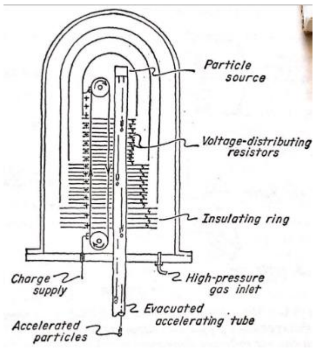


FIG. 9. *The Cockroft-Walton accelerating tube is shown in cross section in this diagram. The intermediate electrodes accelerate the ion beam on its journey down the tube. Since the hydrogen ions (protons) are positively charged, they are attracted to the negative terminal at the bottom of the tube.*



8. The Van de Graff generator can charge its dome up to 7 MV and is used to accelerate ions. Supposing these to be protons obtained by ionizing hydrogen gas, do they reach relativistic speeds? What speeds do they reach? What is v/c exactly? What is v/c classically? What is the % difference?



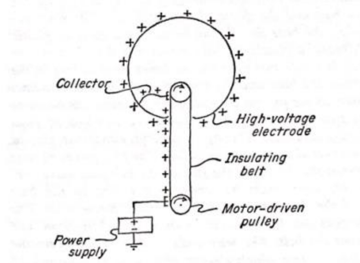


FIG. 12. *Workings of an actual Van de Graaff generator are shown schematically in the simplified diagram at the top. The bottom diagram shows the flow of electric charge when the machine is in operation. Note that it is distributed all over the spherical dome.*

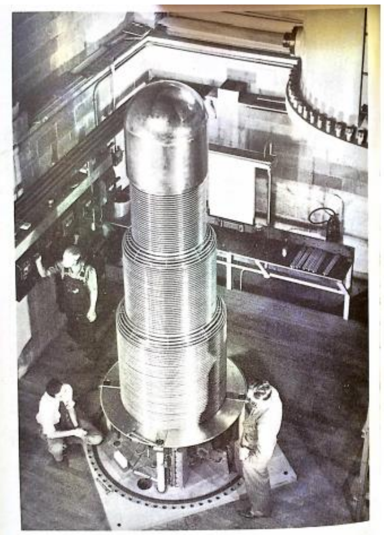


PLATE IV. Large (5-Mev) Van de Graaff generator is shown here with its pressure cover (upper right) removed. Note the polish of the high-voltage terminal at top. Rings around the column are voltage dividers. In this M.I.T. installation the laboratory for experiments with beams is below the machine.

9. The tandem van de Graff gives twice the energy to ions as does the simple van de Graff. If the electrostatic potential achieved is 7 MV, what is the velocity that protons achieve. What is v/c exactly? What is v/c classically? What is the % difference?

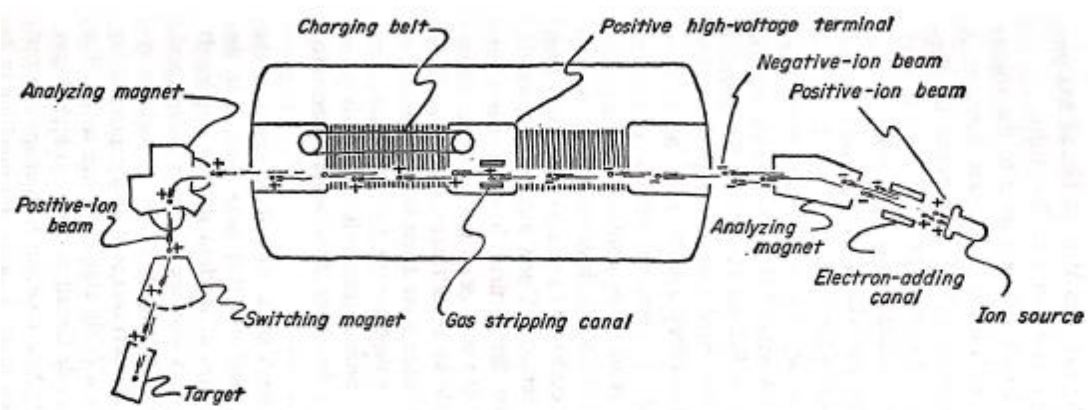


FIG. 13. *The tandem Van de Graaff changes a negative-ion beam into a positive-ion beam to accomplish a double acceleration. Positive ions from the ion source at the right pick up electrons in the electron-adding canal and, passing through the analyzing magnet, form a negative-ion beam of ions of the correct mass and energy. In the main vacuum tube the negative ions are accelerated to the terminal, which is kept at a high positive voltage by the charging-belt system. Passing into a second canal, the beam loses electrons in the presence of a gas and emerges as a positive-ion beam accelerated away from the high positive electrode.*

10. In the linear particle accelerator (LINAC), the acceleration happens in the space between accelerating tubes, which are oppositely biased by being 180 deg out of phase in the applied AC. By accelerating the particles in a series of small steps, high energies can be achieved without high voltages. Particles travel at constant velocity inside the drift tubes. How many sections are needed to achieve a final

proton energy of 20 MeV if the amplitude of the 20 MHz applied AC is 500 kV? What is the length of the final tube, and how much longer is it than the one previous?

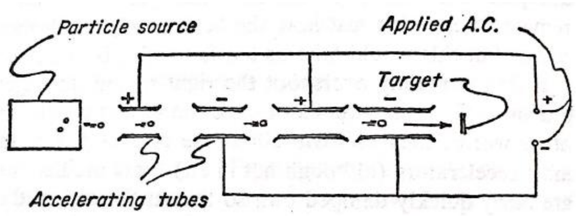


FIG. 17. *The arrangement of accelerating electrodes in a linac is shown schematically in this diagram, in which the particle path extends from the source at the left to the target at the right. Notice that each electrode is longer than the one before it, and that when one electrode is positive, the two on either side are negative.*

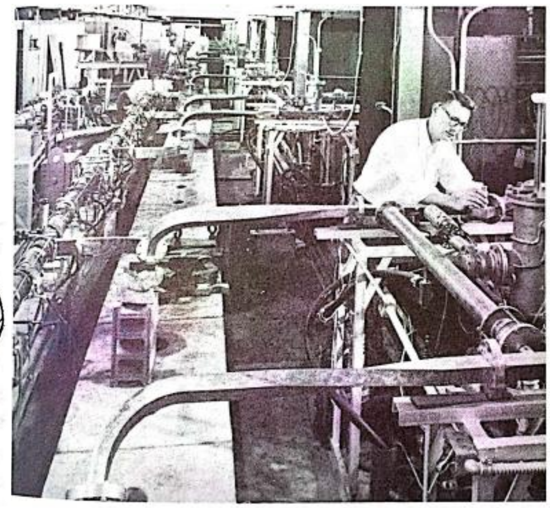


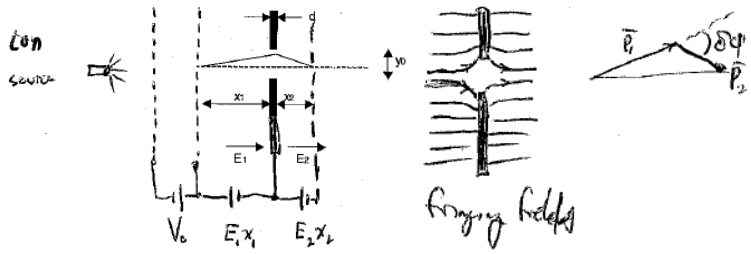
PLATE V. The size of the Stanford University linac is suggested in this view of a small section of the machine. Accelerating tube is at the left. The crosswise tubes installed at intervals along the line are waveguides to conduct radio-frequency power to the tube.

11. An early Stanford LINAC accelerates electrons to 600 MeV. What velocity do the electrons achieve? How many sections do we need now if the voltage amplitude is 500 kV? What is the length of the final tube, and how much longer is it than the previous one? The more recent Stanford LINAC accelerates electrons to 50 BeV. What would be the most obvious design difference with the earlier one?

Section 20.

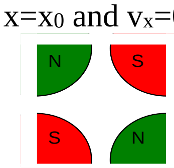
1. Assuming an electron starts from rest, how long does it take for it to become relativistic (say v = 0.9 c) in an E-field of 100 kV/m? How far will the electron have traveled during this time? (Hint: add constant of integration to assure initial condition is satisfied.) How far would the electron have gone according to classical physics?
2. Consider an electron starting from y = - infinity with momentum p_0. For y>0 it encounters a uniform electric field $\mathbf{E} = 100$ kV/m $\mathbf{e}_x$. Plot and compare exact and classical trajectories for p_0 = 10 mc, mc, and mc/10 for distances traveled in the x direction for which v_x remains below 0.9 c (see previous problem). Comment on the differences in each case.
3. A lens for focusing a beam of ions is shown in the figure. It consists of a slit in a metal plate of thickness d. The slit is long in comparison with its height y_0. It separates a region in which the electric field is E_1 from a region in which the electric field is E_2. An ion beam originating from a focus at a distance x_1 to the left of the lens is refocused at a distance x_2 to the right, where x_1 and $x_2 >> y_0$. The voltage through which the ions were accelerated before reaching the lens is $V_0 >> E_1x_1$ and E_2x_2, which allows us to take the trajectory as a straight line except inside the slit. Note that by symmetry, the transverse field is zero at the center of the slit y=0. (a) Find the transverse electric field E_y near the center of

the slit using div(**E**)=0 and approximating E_y as linear in y. (b) Find the transverse force F_y acting on an ion of charge e that enters the slit at height y, determine the net impulse Δp given the ion, and find the deflection angle $\delta\phi = \Delta p/p$. (c) Show that for non-relativistic ions, $1/x_1 + 1/x_2 = (E_2-E_1)/2V_0$, which is a thin lens equation for ion beam focusing.

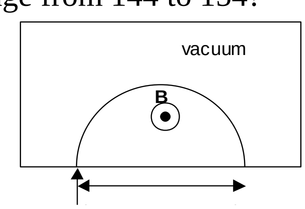


Section 21.

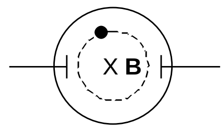
1. Consider a proton with kinetic energy 10 MeV (which does not include the proton's rest energy) in a magnetic field of 1 T. Is the proton relativistic? Calculate the cyclotron frequency and the radius of the orbit. Repeat the calculation for an electron with energy 50 GeV, such as are produced at CERN.
2. A magnetic quadrupole field can be used as a focusing field for a charged particle beam. The cross section of the pole faces is shown in the figure. The pole faces are hyperbolas of the form xy = constant. There are two north poles and two south poles, marked on the figure. The dimension of the magnet perpendicular to the cross section is *l*. The magnetic field in the region $0<z<l$ is $\mathbf{H}(x,y,z)=h(y\,\mathbf{e}_x + x\,\mathbf{e}_y)$, in which $h > 0$; the field is 0 for $z<0$ and $z>l$. Particles enter from negative z with velocity $\mathbf{v_0} = v_0\,\mathbf{e}_z$ and are deflected by the force $\mathbf{F} = (e/c)\,\mathbf{v_0} \times \mathbf{H}$. Neglect the small components v_x and v_y in calculating the force. (a) Sketch the **H** field lines in the xy plane. (b) Explain qualitatively why **H** gives focusing in the **x** direction and defocusing in the **y** direction, assuming the beam particles are positively charged. (c) Write the equations of motion for a beam particle with charge e and mass m, using the approximate force given above. Solve for x as a function of z for z>0, assuming $x=x_0$ and $v_x=0$ at z = 0. Sketch a graph of x (z).



3. The figure shows the essential features of an early mass spectrograph of A. Dempster. Singly positive ions enter the vacuum chamber vertically through the slit, after having been accelerated through a voltage of 20.0 kV. Their paths are bent by the magnetic field **B** and they are deposited a distance s from the slit on a photographic plate. (a) If s = 25 cm for ions of Samarium with mass number 150, i.e., $^{150}Sm_{62}$, what is B? (b)What is the range of s for the stable isotopes of Sm, whose mass numbers range from 144 to 154?

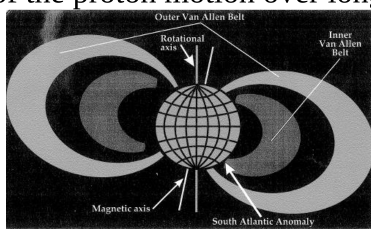


4. The magnetron is a vacuum-tube device that is used to generate ultra-high frequency currents in microwave sources, like microwave ovens or radar transmitters. The frequency range is 10^9 Hz to 10^{11} Hz. A schematic design for a magnetron is shown in the Figure. An electron bunch circulates in a constant magnetic field B, passing electrodes at opposite ends of a diameter of the orbit. The potential V at either electrode oscillates with the distance from the electron bunch. (a) Determine the frequency of the alternating potential. (b) Determine B for a microwave frequency of 10^{10}Hz.



5. A beam of hydrogen isotopes enters a mass spectrometer. The protons and deuterons have been accelerated from rest by a potential drop V_0. The radius of the proton orbit is 10 cm. Calculate the radius of the deuteron orbit.
6. The equations of motion of a charge e in a magnetic field H_0 $\mathbf{e}_z$ are $dx/dt=v_x$, $dy/dt=v_y$, $dv_x/dt=\omega v_y$, $dv_y/dt=-\omega v_x$ where $\omega = eH_0/m$ in SI units. Solve the equations numerically on a computer. Set $\omega=1$ and take initial values $(x_0,y_0,v_{0x},v_{0y})=(1,0,0,1)$. You might integrate the equations stepwise for a small time step. Or, more simply, use an analytic computer program with a built-in differential equation solver. Plot the trajectory, i.e. x(t), y(t) as a function of t. It should be a circle.
7. The typical trajectory of a charged particle in a uniform magnetic field is cyclotron motion. The magnetic force pointing toward the center F = e (v/c) H provides the centripetal force, but this does NOT equal m v^2/R as in classical mechanics. (a) What does it equal in terms of p, v, and R? (b)Show that the classical expression is recovered when $v<<c$. (c) Find the momentum p in terms of e, H and R. (d) How does it compare to the classical expression?
8. An ion moves in a helical path around the axis of a long solenoid wound so that the ion encounters a region in which the field intensity increases gradually from H_1 to H_2. Derive the condition that the ion will be reflected somewhere in terms of H_1, H_2, and the longitudinal and transverse momentum components.
9. Determine the frequency of vibration of a charged spatial oscillator, placed in a constant uniform magnetic field; the proper frequency of vibration of the oscillator (in the absense of the field) is ω_0. Estimate the change (absolute and relative) in the vibrational frequency of the CO_2 molecule (2350 cm^{-1}) due to a magnetic field of 1 T. (In S.I. units the cyclotron frequency (21.8) becomes eB/m.) What spectral resolving power $\lambda/\Delta\lambda$ would be needed to observe your estimated shift, and is this available in commercial infrared spectrometers?
10. The inner van Allen belt traps a lot of cosmic ray protons, and even some antiprotons, as discovered recently by the satellite-borne spectrometer PAMELA (see Physics Today Oct 2011). Trapped protons have energies up to a few GeV. Is a 3 GeV proton relativistic? The altitude of the inner van Allen belt is 10000km at the equator. What is the magnitude of the Earth's magnetic field there if the field surface-field at the equator is 0.5 G (see section 44 for dipole field)? If the angle between the proton's momentum and the local magnetic field

lines at the equator is close to 90 deg, what is the frequency of the helical motion in Hz for a 3 GeV proton? What is the radius of the helical motion? What is the radius for a 1 GeV proton? PAMELA accesses the inner van Allen belt only near the magnetic pole at an altitude of ~600 km. What is the magnetic field there? What is the radius of helical motion in this region for 1 GeV protons? Describe qualitatively the nature of the proton motion over long times.



11. A proton of velocity 10^7 m/s is projected at right angles to a uniform magnetic induction field B of 0.1 T. How much is the particle path deflected from a straight line after it has traversed a distance of 1 cm? How long does it take for the proton to traverse a 90 deg arc?
12. See Post's 1958 paper on the pyrotron. The adiabatic invariant is stated to be the magnetic moment associated with the circular motion of the particle. Post considers non-relativistic motion. (a) From these considerations derive Post's Eq. (1). (b) Show that Eq (21.9) in LL2, if $v \ll c$, gives the invariance of the magnetic moment that Post claims. (c) Derive Post's Eq. (3).

Section 22.

1. (a)Use a computer program to plot the cycloid curve in the xy plane given by x = t-Sin[t], y = 1-Cos[t]. (b) For a charge e moving in orthogonal E and H fields, starting from rest at the origin, plot the kinetic energy as a function of time. (c) Plot also modified functions as in (a) that give trochoid curves as in section 22, Fig. 6 a&b.
2. The experiment by which Thomson discovered the electron consisted of a cathode ray passing between parallel capacitor plates in a uniform magnetic field. The electrons travel parallel to the plates and B is perpendicular to both E and v. Derive the condition (in S.I. units) relating the potential difference V_0 between the plates and the magnetic field strength, along with any other relevant parameters, such that the cathode ray is undeflected, assuming the cathode ray is a beam of electrons. This is the principle of the velocity selector in a mass spectrometer. A Bainbridge mass spectrometer includes a velocity selector in the vacuum chamber through which the positive ions pass. The selector uses a horizontal E-field and the same B field that bends the path of the ions in the spectrograph proper. What is the mass of the ions that impinge on the photographic plate a distance s from the slit?
3. The equations of motion (SI units) of a charge e in crossed electric and magnetic fields $E_0\,\mathbf{e}_y$ and $B_0\,\mathbf{e}_z$ for motion with $v_z=0$ are $dx/dt=v_x$, $dy/dt=v_y$, $dx/dt=\omega v_y$, $dv_y/dt=a-\omega v_x$, where $\omega=e\,B_0/m$ and $a = e\,E_0/m$. Solve the equations numerically on a computer. For illustration purposes choose units with $\omega=1$ and $a=1$. If the particle starts at rest at the origin the trajectory is a cycloid. Explore what

happens for different initial values of v_{0x}, both positive and negative, keeping $v_{0y}=0$. Explain why if $v_{0x} = 1$ (in these units) the particle moves in a straight line.

4. (This is Landau problem 1.) Determine the relativistic motion of a charge in parallel uniform electric and magnetic fields. Sketch the motion and discuss qualitatively. (This is the configuration of fields and characteristic motion of relativistic electrons in a vacuum gyrotron, a high power microwave source.)

Section 23

1. Determine the components of the electromagnetic field tensor from its definition Eq. (23.3).
2. Show that the space components for the equation of motion (23.4) give the relativistic Lorentz force law. Show that the time component of the equation of motion (23.4) gives the work equation.
3. Find the tensor $\mathcal{F}^{ik} \equiv (1/2)\, e^{iklm}F_{lm}$ that is dual to the electromagnetic field tensor F^{ik}.
4. Show that $u^i F_i{}^k u_k = 0$.
5. Complete the entries in the electromagnetic field tensor below. What scalar is found from the product of the electromagnetic field tensor with itself, i.e. $F_{ik}F^{ik}$=? How does this change when one changes between different inertial reference frames?

$$F^{ik} = \begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ & & -H_z & H_y \\ & & & -H_x \\ & & & \end{bmatrix}$$

6. Determine the relationship between the Lorentz force law and the energy equation, starting from the 4-D equation of motion.

Section 24

1. Derive the Lorentz transforms for the components of **E** and **H** equations (24.2) and (24.3). What are the Lorentz transformations of the fields in SI units? Verify their vector form (24.4) when $V \ll c$.
2. Derive the relation (24.5) between **H** and **E** when **H**' = 0 in the K' system. Derive the relation (24.6) between **E** and **H** when **E**' = 0 in the K' system.
3. Show that if **E** ⊥ **H** are perpendicular in some system K and E>H, that there exists a system K' where the field is purely electric and the velocity of that system relative to K satisfies **V**⊥**E**, **V**⊥**H**, and V=cH/E. Show that if E⊥H in some system K, and E<H, there exists a system K' where the field is pure magnetic, with **V**⊥**E**, **V**⊥**H**, and V=cE/H.
4. An observer at rest in inertial frame K finds himself to be in an electric field E = (0,E,0) with no magnetic field. An observer in K' moving along the common X, X' axes with speed V finds electric and magnetic fields **E**' and **H**'. Find these fields and show that **H**' = (-1/c) **V** x **E**', which is (24.5) with **V**→**-V**.
5. Two large non-conducting parallel plates, separated by a distance d and oriented perpendicular to the Z axis move together along the X axis with

velocity V, which is not necessarily small compared with c. The upper and lower plates have uniform surface charge densities $+\sigma$ and $-\sigma$ respectively in the rest frame K' of the plates. Find the magnitude and direction of the electric and magnetic fields between the plates according to an observer in the lab frame K (neglecting edge effects).

6. In the lab, an electron moves with constant velocity $\boldsymbol{v}$ in crossed $\boldsymbol{E}$ and $\boldsymbol{H}$ fields. What are the fields and force on the electron in its rest frame?

Section 25

1. What scalar is found from the product of the electromagnetic field tensor with itself, i.e. $F_{ik}F^{ik}$=?
2. Show that the product of the electromagnetic field tensor F_{lm} with its dual (1/2) $e^{iklm}F_{ik}$ is a 4-divergence.
3. What pseudoscalar is found from the product of the electromagnetic field tensor with its dual tensor, i.e. $F^{*lm} F_{lm}$=?
4. Derive the Lorentz transform for the complex vector **F** = **E** + i**H** in the form (25.6).
5. The axes of frame K' are aligned with those of frame K. K' moves with respect to K along the common X,X' axis with speed V. In K there is a uniform electric field **E**=[0,E,0] and a uniform magnetic field **H**=[0,0,H]. Show that it is possible to choose H so that the field in K' is entirely magnetic with magnitude $\sqrt{(H^2-E^2)}$. What is the direction of **H**' in K'? What is the relation between the chosen H value and the values of E & V?
6. Determine the velocity of the system of reference in which the electric field **E** and magnetic field **H** are parallel. In the lab you connect a battery to some parallel plates, then stick these between the poles of a magnet, such that the field strengths are E=1V/cm and B=0.1 T, while the field vectors are at 30 degrees with respect to each other. How fast do you have to run to make the fields appear parallel to you? (To convert your formula from the first part to SI units, make the substitution H→cB.)
7. Show that the equation **F** = **E** + i **H** = a **n**, where **E** and **H** are given, a is complex, and **n** is a complex unit vector, gives 8 equations for 8 unknowns.
8. If = **F** = **E** + i **H** = a **n** = (a' + i a")(**n**' + i **n**"), and by choice of coordinates **n** is real, show that **E** and **H** are parallel.

Section 26.

1. (a) Show from the definition of the electromagnetic field tensor that $Z_{ikl} \equiv \partial F_{ik}/\partial x^l + \partial F_{kl}/\partial x^i + \partial F_{li}/\partial x^k = 0$. (b) Show that Z_{ikl} is antisymmetric in all three indices. (c) Show that Z_{ikl} is non-zero only when all three indices are different. (d) What 4 equations for **E** and **H** components are obtained by setting i=0,1,2,3 in $Z_{ikl}=0$? (e) The 4-vector $e^{iklm}\partial F_{lm}/\partial x^k = 0$ is dual to the rank 3 tensor $Z_{ikl} = 0$ and gives the same Maxwell equations (26.1) and (26.2), i.e. the first two Maxwell equations. Find the equation results from setting i=1.
2. There is a time dependent current $I_s(t)$ in a long and densely wound solenoid. (a) Determine the electric field at radius r on the midplane of the solenoid, both inside and outside the solenoid. (Hint: The direction of E is azimuthal; use the

Amperian loop trick.) (b) From your result of (a) calculate the curl of **E** at radius r.

3. A long straight wire carries an alternating current $I(t) = I_0 \mathrm{Cos}[\omega t]$. Nearby is a square loop. The wire lies in the plane of the loop, parallel to two sides of the square, which are at distances a and b from the wire. (The side of the square is b-a.) Determine the current induced in the square loop if its resistance is R.
4. A circular loop of wire with radius a and electrical resistance R lies in the xy plane. A uniform magnetic field is turned on at time t=0; for t > 0 the field is $H(t)=(H_0/\mathrm{Sqrt}[2])\,(\mathbf{e_y}+\mathbf{e_z})\,(1-\mathrm{Exp}[-\lambda t])$. (a) Determine the current I(t) induced in the loop. (b) Sketch a graph of I(t) versus t.
5. A metal disk of radius a, thickness d, and conductivity σ is located in the xy plane, centered at the origin. $\mathbf{J}=\sigma\,\mathbf{E}$. There is a time dependent uniform magnetic field $\mathbf{B}(t)=B(t)\,\mathbf{e_z}$. Determine the induced current density $\mathbf{J}(x,t)$ in the disk.

Section 27

1. The Lagrangian for a system of particles is the sum of individual Lagrangians: $L=\Sigma_a L_a$. If particles are replaced by mass and charge densities, then the Lagrangian is replaced by an integral over Lagrangian density, $L(t) = \int \Lambda(r,t)\, d^3r$. The trajectories of individual particles are replaced by the velocity field for the continuous matter filling the space. Variation of trajectory is replaced by variation of velocity field. This idea holds also for E-M fields. Identify the Lagrangian density D for the E-M field.
2. The action for the E-M field is $S=(1/c)\int D\, d\Omega$, where $d\Omega$= the differential volume in 4-space, which is a Lorentz invariant scalar (see eq. 6.13). S is also an invariant scalar (see section 8). Therefore, D must be a scalar, and from section 27 it must be quadratic in the fields. We can form such a scalar from the 4-potential and its derivatives as A^iA_i, $F^{ik}F_{ik}$, and $(1/2)\, e^{iklm}F_{lm}F_{ik}$. What are these three scalars in terms of the potential or field components? (The first and last terms are excluded from D. Exclusion of the last term has two reasons: It is a psuedoscalar, while both S and $d\Omega$ are true scalars. And, it is a 4-divergence (see section 25 Problem 3), so when integrated it gives just a constant, which when varied gives zero. Hence it contributes nothing when finding the field equations from Hamilton's principle.)

Section 28

1. A solid sphere of radius *a* has total charge Q uniformly distributed throughout its volume. The sphere rotates with angular velocity $\omega = \omega\, \mathbf{e_z}$. Find the current density $\mathbf{J}(\mathbf{r})$. Use spherical polar coordinates.
2. Determine the Lorentz transformation of the charge density ρ and current density $\mathbf{j}$. If $\mathbf{j}'=0$ but ρ' is nonzero in frame K', which moves at speed V relative to frame K, what are ρ and $\mathbf{j}$ in frame K? Show that $\mathbf{j}=\rho\mathbf{V}$. Explain why ρ is greater than ρ'.
3. Why must the electron be a stable fundamental particle? See G. Feinberg and M. Goldhaber, Proc. Natl Acad. Sci. USA 45, 1201 (1959). Hint: What particles are lighter than electrons and what are their charges?

Section 29

1. Show that setting the 4-divergence of current four vector equal to zero gives the equation of continuity.
2. Show that due to charge conservation, the gauge transformation $A_i \rightarrow A_i - \partial f/\partial x^i$ has no effect on the equation of motion for a charge in given fields.

Section 30

1. A capacitor with circular parallel plates, with radius a and separation d, has potential difference V(t). (a) Determine the magnetic field on the midplane of the capacitor, at radius r from the symmetry axis, for r>a, in terms of dV/dt, in both Gaussian and SI units. (b) Show that H is the same as the field of a straight wire carrying current I=dQ/dt, where Q is the charge on the capacitor.
2. Show that the discontinuity of B across a capacitor plate as the capacitor is being charged with current I is (in SI units) equal to μ_0 **K x n** where $\mathbf{K} = \mathbf{e}_r I (1/r - r/a^2)/(2\pi)$. Determine the surface charge density on the capacitor plates (assumed circular) as a function of time, and comment on the radial distribution of charge.
3. Deduce the equation of continuity from the 2nd pair of Maxwell equations, both in 3-D and 4-D form.
4. A cylindrical non-magnetic wire, radius R, carries a uniform steady current I. Find **H** inside and outside the wire. If the current is 30 kA, what is the field in T at a distance of 1 m?
5. A long non-magnetic cylindrical conductor, inner radius *a*, outer radius *b*, carries a uniform current I. Find the magnetic field **H**(r) inside the hollow space, within the conductor, and outside the conductor.
6. Three straight, co-planar, infinitely long, equally spaced wires (with zero radius) each carry a current I in the same direction. Calculate the location of the two zeros in the magnetic field. Sketch the magnetic field line pattern.

Section 31

1. Consider a capacitor with circular plates of radius *a*, which is charging with current *I*. What is the energy flux density on the cylindrical surface of radius *a* that encloses the volume between the plates? What direction is electromagnetic energy flowing through this mathematical surface? What is the total rate at which field energy flows into the cylindrical volume? Show that this equals the rate of change of the total electric field energy stored between the capacitor plates.
2. Consider a segment of conducting wire that extends from z = 0 to z = *l*. The power supplied to the moving charges in the dc current *I*, by the potential difference V_0 from one end of the segment to the other, is $P = I V_0$. Show that this equals the electromagnetic power that flows in through the surface of the wire.
3. Prove by using tensor methods that div(**E**x**H**) = **H**.curl**E** - **E**.curl**H**.
4. If W is the energy density and **S** is the Poynting vector for an electromagnetic field, prove that $c^2W^2 - S^2$ is invariant.

Section 32.

1. Show that Λ in (32.1) is the Lagrangian density.

2. What are the three four-vectors dx^i, dx'^i, and dx''^i that span the hyperplane x^0=constant? Show that $dS^1 = dS^2 = dS^3 = 0$ when integrations are carried out over the hypersurface x^0=const. Show that $\int T^{0k} dS^k = \int T^{00} dV$ if the integration is carried out over the hyper surface x^0=const.
3. Consider the rank 3 tensor $\psi^{ikl} = -\psi^{ilk}$. Show that $\partial^2 \psi^{ikl}/\partial x^k \partial x^l = 0$. Show that the definition of the energy-momentum tensor T^{ik} (32.3) is not unique, i.e. one can always obtain another tensor T'^{ik} by adding a term $\partial \psi^{ikl}/\partial x^l$ to T^{ik}, such that T'^{ki} also satisfies the conservation law (32.4). Show that $\int (\partial \psi^{ikl}/\partial x^l) dS^k = (1/2) \int (dS_k \, \partial \psi^{ikl}/\partial x^l - dS_l \, \partial \psi^{ikl}/\partial x^k)$. Show that T'^{ik} gives the same momentum 4-vector, i.e. that the momentum 4-vector is uniquely determined even though the energy-momentum tensor is not.

Section 33.

1. Show that the energy-momentum tensor for the electromagnetic field (33.1) is symmetric. Show that the trace of the energy momentum tensor for the electromagnetic field is zero.
2. Show that T^{00} is the electromagnetic energy density. Show that $cT^{0\alpha}$ are the components of the Poynting vector. Derive the components of the Maxwell stress tensor.
3. (a) If the electric and magnetic fields are parallel, or if the electric field is zero, or if the magnetic field is zero, when the field is parallel to the x axis, show that the energy-momentum tensor is diagonal with components W, -W, W, W, where W is the energy density.
4. If the electric and magnetic field magnitudes are equal and the electric and magnetic fields are perpendicular (parallel to x and y axes, respectively), show that the non-zero components of the electromagnetic field tensor equal W and occur only in the "corners" of the matrix.
5. (a) Show that for non-interacting paricles, $T^{0\alpha} = \mu c^2 u^\alpha$, where $\alpha = 1,2,3$, μ is the mass density, and u^i is the velocity four-vector. (b) Show that $T^{0\alpha} = T^{\alpha 0} = \mu c u^\alpha u^0 ds/dt$. (c) Use the physical interpretation of T^{00} to show it equals $\mu c u^0 u^0 ds/dt$. (d) Use the physical meaning of the Maxwell stress tensor to show that $T^{\alpha\beta} = \mu c u^\alpha u^\beta ds/dt$, $\alpha=1,2,3$.
6. Consider an infinite parallel-plate capacitor, with the lower plate (at z = -d/2) carrying the charge density $-\sigma$, and the upper plate (at z = +d/2) carrying the charge density $+\sigma$. Determine all nine elements of the Maxwell stress tensor, in the region between the plates. Display your answer as a 3 x 3 matrix. Determine the force per unit area on the top plate. What is the momentum per unit area, per unit time, crossing the xy plane? At the plates this momentum is absorbed, and the plates recoil unless fixed by external force. Find the recoil force per unit area on the top plate.
7. Fill in the details in the derivation of (33.7) for the 4-div of the energy momentum tensor of the electromagnetic field and of (33.9) for the 4-div of the energy momentum tensor of the particles.
8. Determine the net force on the northern hemisphere of a uniformly charged solid sphere of radius R and charge Q using the Maxwell stress tensor.

9. Consider two equal point charges q, separated by a distance 2a. Construct the plane equidistant from the two charges. By integrating Maxwell's stress tensor over this plane, determine the force of one charge on the other. Do the same for charges that are opposite in sign.
10. A charged parallel-plate capacitor with uniform electric field $\mathbf{E} = E\,\mathbf{e}_z$ is placed in a uniform magnetic field $\mathbf{H} = H\,\mathbf{e}_x$. Find the electromagnetic momentum in the space between the plates. Now a resistive wire is connected between the plates, along the z axis, so that the capacitor slowly discharges. The current through the wire will experience a magnetic force; what is the total impulse $\Delta p=\int F dt$ delivered to the system during the discharge? Instead of turning off the electric field, suppose we slowly reduce the magnetic field. This will induce a Faraday electric field, which in turn exerts a force on the plates. Show that the total impulse is again eqaual to the momentum originally stored in the fields.
11. A uniform constant magnetic field H is directed along the z-axis of an inertial frame. Find the energy momentum tensor in both Gaussian and S.I. units.

Section 35

1. A straight rod has cross-sectional area A and mass m per unit length. It lies along the X-axis of an inertial frame in a state of tension F. (a) Show that the energy-momentum tensor has components which are the elements of a 4 x 4 diagonal matrix, with diagonal elements (mc^2/A, $-F/A$, 0, 0). (b) Deduce that an observer moving along the X-axis with speed V observes an energy per unit length of the rod to be $(mc^2 - FV^2/c^2)/(1-V^2/c^2)$. Deduce that F cannot exceed mc^2.

Section 36

1. A long uniformly charged ribbon is located in the xz plane, parallel to the z axis, occupying the region $-\infty \le z \le \infty$ and $-a/2 \le x \le a/2$. The charge per unit area on the ribbon is σ. Determine $\mathbf{E}$ at (x, 0, 0), where $x > a/2$. What is the asymptotic field on the x axis? Determine $\mathbf{E}$ at (0, y, 0), where $y > 0$. What is the asymptotic field on the y axis?
2. A charge +q is at (0, 0, z_0). What is the electric field $\mathbf{E}(\mathbf{r})$?
3. Find the restrictions on C_1 and C_2 such that the function $V(r,\theta) = (C_1 \text{Cos}^2[\theta] + C_2)/r^3$ may be a potential function in a charge-free region of space.
4. A semi-infinite wire lies on the negative z axis, from z = 0 to $z = -\infty$, with constant linear charge density λ. Determine $\mathbf{E}$ at a point (0, 0, z) on the positive z axis. Determine $\mathbf{E}$ at a point (x, 0, 0) on the positive x axis.
5. For any charge free region of space, prove the mean value theorem: If S is the surface of a mathematical sphere whose interior contains no charge, then the potential at the center is equal to the average of the potential over the surface S.

Section 37

1. Calculate the energy per unit length for two long coaxial cylindrical shells, neglecting end effects. The inner and outer cylinders have radii *a* and *b*, and linear charge densities λ and $-\lambda$, uniformly distributed on the surface, respectively.

2. Consider two charges separated by a distance *a*. If $\mathbf{E}_1$ is the field of q_1 and $\mathbf{E}_2$ is that of q_2, find the interaction part U_{12} of the potential energy. (Hint: U_{12} depends only on *a*, so without loss of generality let q_1 be at the origin, and q_2 on the z axis at z = *a*.)
3. Find the electrostatic energy of a uniformly charged solid sphere with total charge Q and radius R. Use the result to compute the electrostatic energy of an atomic nucleus (charge = Ze, radius = (1.2 x 10^{-15} m) $A^{1/3}$] in MeV times $Z^2/A^{1/3}$. Calculate the change of electrostatic energy when a uranium nucleus (Z = 92, A = 238) fissions into two equal fragments.
4. Calculate the self-energy of a charged spherical surface, with total charge Q uniformly distributed on the surface, and radius R. What is the physical significance of this result? What is the limit of the self-energy as $R \to 0$?
5. Let us assume that an electron is a uniformly charged, spherical particle of radius R. Determine the self energy of the distribution. Assuming that the rest energy is electrostatic in origin, determine the "classical radius" R. Calculate a value in meters and compare to the size of a typical nucleus.

Section 38

1. Complete the derivation of (38.6) for the electric field of a uniformly moving charge.
2. For velocities v of a charged particle close to c, show that the half width at half maximum of the angular distribution of the electric field (38.8) is $\Delta\theta \approx 0.8 \sqrt{(1-v^2/c^2)}$.
3. Show that the fields of a moving charge satisfy Maxwell's equations, apart from the singularity at $\mathbf{r}$ = [vt, 0, 0].
4. A proton at the origin exerts a force on the nucleus of a gold atom at rest on the z axis at z = 100 fm. (The atomic electrons are irrelevant.) Compute the force on the Au nucleus if the proton is (a) at rest, and (b) moving along the x axis with speed 0.99 c. Also, in the latter case, plot the magnitude of the force on the Au nucleus as a function of time. In making the plot it is convenient to measure time in zeptoseconds (zs) where 1 zs = 10^{-21} s. Express the forces in MeV/fm. (Neglect the motion of the heavy Au nucleus.)
5. For a charge in uniform motion with velocity v, show that $R^* = R \sqrt{(1-(v/c)^2 \mathrm{Sin}^2\theta)}$, where R is the vector from the present position of the charge to the field point and $\mathbf{R}.\mathbf{v} = R\, v \cos\theta$. Plot a polar graph of |E| as a function of θ for a fixed R, assuming v/c = 0.8.
6. Determine the force (in the lab frame) between two charges moving with the same velocity **V**. What is the ratio of transverse-to-longitudinal force? Explain what happens to this ratio when the two charges lie on a line parallel to **V** or perpendicular to **V**?

Section 40

1. Complete the derivation (40.8) for the electric field of a dipole, i.e. take the necessary gradients. Show that (40.8) and (40.9) are equivalent.

2. Derive (40.10) and (40.11) for the Cartesian and spherical coordinate components of the electric dipole field in a plane passing through **d.** (My edition has a sign typo!).
3. Three charged line segments, each with linear charge density λ, extend from the origin O to (a, 0, 0), from O to (0, b, 0), and from O to (0, 0, c). Find the dipole moment of this charge distribution. Find the first two terms in the multipole expansion of the potential on the z axis for z >> a,b,c. What are the monopole and dipole contributions to **E**(0, 0, z) for z >> a, b, c?
4. A point dipole $\mathbf{p} = p\,\mathbf{e}_z$ is at the origin. At point $P_1 = (x_1,0,0)$ there is a point charge e. What is the force on this charge due to the field of the dipole? What is the force on the dipole? How much work is required to take the charge from P_1 to infinity if the dipole remains fixed at the origin? How much work is required to take the dipole from the origin to infinity if the charge remains fixed at P_1? What is the physics underlying the simple answers to these questions?
5. A point dipole $\mathbf{p} = p\,\mathbf{e}_z$ is at the origin. A point charge e is located at $P_2 = (x_2, y_2, z_2)$. How much work is required to move the charge? What is the force on e when it is at P_2?
6. The dipole $\mathbf{p} = [p_1, p_2, p_3]$ is located at the origin. What is the potential in spherical coordinates? What is the electric field in spherical coordinates?
7. A point dipole $\mathbf{p} = p_0\,\mathbf{e}_z$ is at the origin. A second dipole $\mathbf{p} = p_0\,\mathbf{e}_z$ is at $(0, 0, z_0)$. What is the force on the second dipole? What is the interaction energy? [Hint: The energy is the work required to bring the second dipole to $(0, 0, z_0)$ with the first dipole fixed. Show that the energy is $-\,\mathbf{p}.\,\mathbf{E}(0,0,z_0)$.
8. For a charge distribution with unequatl amounts of positive and negative charge, such as an ion, is the dipole moment vector necessarily directed along the line between positive and negative charge centers?

Section 41

1. Show that $(\partial^2/\partial X_\alpha \partial X_\beta)(1/R_0) = 3\, X_\alpha X_\beta/R_0^5 - \delta_{\alpha\beta}/R_0^3$. Derive the form (41.6) for the quadrupole term of the potential starting from (41.5).
2. Show that if $D_{xx} = D_{yy} = -(1/2)\, D_{zz} = -(1/2)\, D$ holds for the components of the quadropole moment tensor, then the quadropole term of the potential has the form (41.8).
3. If the total charge $\Sigma e = 0$, and the dipole moment vector $\mathbf{d} = \Sigma \mathbf{r} e = 0$, show that the quadrupole moment tensor $D_{\alpha\beta}$ is independent of the choice of origin.
4. Derive the expression for the l^{th} term $\phi^{(l)}$ (41.12) in the expansion of the potential and the 2^l-pole moment $Q_m^{(l)}$ (41.13) from the expansion (41.9) of $1/|R_0-r|$ in spherical harmonics.
5. Using a table of Y_{lm} functions, derive the relations between 2^1-pole moment $Q^{(1)}{}_m$ and the components of the dipole moment **d** (41.14).
6. A linear quadrupole consists of three charges: q, -2q, and q, on the z axis. The positive charges are at z = ±a and the negative charge is at the origin. Show that this system is the same as two dipoles, with dipole moments +q a **e**z and –q a **e**z, centered at z = +a/2 and z = -a/2, respectively. Calculate the potential $\phi(r,\theta)$ in spherical coordinates for r >> a. Sketch the electric field lines in the xz plane. What is the quadrupole moment tensor for this charge distribution?

7. Consider the electric quadrupole consisting of four charges in the xy plane: +q at (x,y) = (*a*,0), -q at (0, *a*), +q at (-*a*, 0), and –q at (0, -*a*). Determine the electric potential on the x axis. What is the asymptotic form of the potential $\phi(x,0,0)$? Use computer graphics to make a log-log plot of ϕ vs. x at verify the asymptotic behavior. Determine the electric field on the x axis for x>>a. Determine the electric field on the z axis.

Section 42

1. Show that $[\nabla(\mathbf{d.E})]_{\mathbf{r}=0} = (\mathbf{d}.\nabla)\mathbf{E}_0$ for the force (42.5) on a system of charges in an external field when the net charge is zero.

Section 43

1. Using the Biot-Savart law, determine the magnetic field due to a current I in a long straight wire.
2. Using the Biot-Savart law, determine the magnetic field due to a circular current loop, at an arbitrary point on the axis of symmetry.
3. A square wire loop of size 2*a* x 2*a* lies in the xy plane with its center at the origin and sides parallel to the x and y axes. A counterclockwise current I runs around the loop. Find the magnetic field on the z axis. Show that for z/*a* >> 1 the field becomes that of a magnetic dipole, and find the magnetic moment. Compare the field at the center of this square loop with that at the center of a circular loop of diameter 2*a*.
4. Consider a circular cylindrical solenoid of finite length *L*, radius *a*, with *N* turns of wire carrying current I_0. The current may be approximated by a surface current density K (=azimuthal current per unti length along the cylinder) equal to NI_0/L. Caclulate the magnetic field on the axis of the cylinder halfway between the ends. (Hint subdivide the solenoid into infinitesimal current rings dI = Kdz'). Calculate the magnetic field on the axis of the cylinder at either end. Show that B_{end}/B_{center} approaches ½ as $L/a \rightarrow \infty$.
5. Use index methods to show that $\nabla \times (f\,\mathbf{a}) = f(\nabla \times \mathbf{a}) + (\nabla f) \times \mathbf{a}$. Show that $(\nabla(1/R)) \times j = (\mathbf{j} \times \mathbf{R})/R^3$.

Section 44

1. Use index methods to show the identity $(\mathbf{r} \times \mathbf{v}) \times \mathbf{R} = (\mathbf{r.R})\mathbf{v} - (\mathbf{v.R})\mathbf{r}$ used to obtain (44.3). Use index methods to derive the identity $\nabla \times (\mathbf{a} \times \mathbf{b}) = (\mathbf{b}.\nabla)\mathbf{a} - (\mathbf{a}.\nabla)\mathbf{b} + \mathbf{a}(\nabla.\mathbf{b}) - \mathbf{b}(\nabla.\mathbf{a})$. Use index methods to show that $\nabla.(\mathbf{R}/R^3) = 0$. Use index methods to show that $(\mathbf{m}.\nabla)(\mathbf{R}/R^3) = (\mathbf{m}/R^3) - 3\mathbf{R}(\mathbf{m.R})/R^5$.
2. A small circular loop carrying current I is in the xy plane with its center fixed at the origin. The dipole moment is in the same direction as $+\mathbf{e}_z$. Sketch the magnetic field lines. A second identical current loop is located on the x axis at some fixed distance *d* from the origin. Assume the axis of the first loop is fixed in the z direction, but the second loop is free to rotate. What is the equilibrium orientation of the second loop? What is the direction of the second dipole moment in stable equilibrium?
3. At the surface of the Earth the magnetic field is approximately the same as the field from a point dipole $\mathbf{m}_E$ at the center of the Earth. The dipole moment is $\mathbf{m}_E$

$= m_E$ [$\sin\theta_0\cos\phi_0$, $\sin\theta_0\sin\phi_0$, $\cos\phi_0$] where $m_E = 7.79 \times 10^{22}$ A m^2, and (θ_0,ϕ_0) = (169 deg, 109 deg). The z axis is the Earth's rotation axis and the x axis passes through the Prime Meridian on which Greenwich lies; positive ϕ is to the east. Calculate the magnetic field H at a point on the earth with colatitude θ and longitude ϕ. Give the components of H to the north ($-\mathbf{e}_\theta$), to the east ($\mathbf{e}_\phi$), and vertical ($\mathbf{e}_r$). Calculate these components for Orlando, in Gauss.

4. The planet Mercury has a dipole magnetic field like Earth's. In 2011, NASA's messenger mission to Mercury discovered that Mercury's dipole is displaced 400 km to the north of center. The radius of Mercury is 2440 km. Do you think this displacement would be easy to detect? What is the ratio of the fields on the surface at north and south magnetic poles? What is the ratio at the altitude of Messenger's closest approach (228 km)? NASA's website gives the surface-field ratio at Mercury's *geographic* poles as 3.5. Discuss any difference with your answer.

Section 45

1. What is the precession frequency for an electron in a magnetic field of 1 T?
2. Proton spins in liquid water become partly polarized in an external **B**-field. When the field is suddenly changed to a new direction perpendicular to the old, the original polarization precesses about the new magnetic field at the Larmor frequency and decays until a new polarization appears parallel to the new **B** direction. The decay happens by interactions with processes that have the same frequency as the Larmor frequency. (a) When the Larmor frequency is ~4 kHz, thermal fluctuations are responsible for the decay. What magnetic field does this correspond to? (b) When the Larmor frequency is ~400 Hz, proton exchange between H_3O^+ and H_2O, and between H_2O and OH^- is responsible for the relaxation. What B does this correspond to? See Phyiscs Today Oct 2011.
3. Look up Marta Anguera Antonana PhD dissertation (UCF 2017). Find the plot of ferromagnetic resonances for a tiny sample of Yttrium Iron Garnet at the center of a microwave strip line resonator. Transmission of microwaves is reduced when the microwave frequency is resonant with the precession of the sample's magnetic moment in an external magnetic field. Do the data satisfy Larmor's formula?

Section 46

1. Show that the second pair of Maxwell's equations in vacuum (46.2) are equivalent to $\partial F^{ik}/\partial x^k=0$.

Section 47

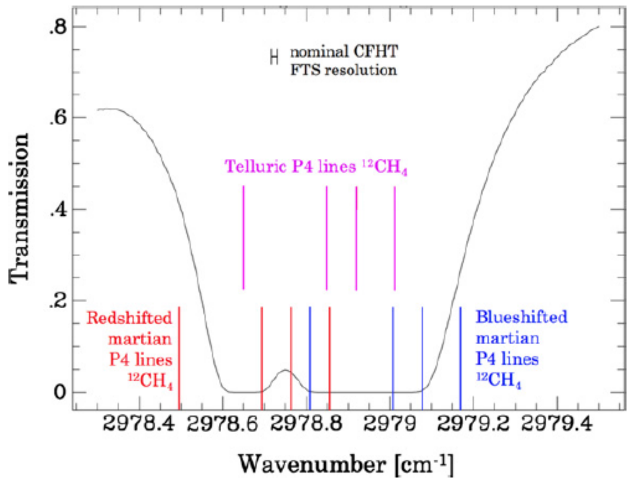
1. Show that if the direction of propagation of a plane wave is along X, then the only non-zero component of the Maxwell stress tensor is $-\sigma_{xx}$ = W.
2. A field meter shows that the amplitude of the electric field oscillation in a certain radio wave is 5 millivolts per meter. What is the amplitude of the magnetic field oscillation in T? What is the intensity in W/m^2?
3. The electric field in an electromagnetic wave is $\mathbf{E}(y,t) = E_0(\{-1,0,1\}/\sqrt{2})\,\mathrm{Sin}(ky-\omega t)$. What is the magnetic field **H**? What is the Poynting vector?

4. Consider the fields $\mathbf{E}=\{F_1,F_2,F_3\}$ and $\mathbf{H}=\{G_1,G_2,G_3\}$, where the Cartesian components of each field are all functions of (x-ct), so that they obviously satisfy Maxwell's equations, and all components $\rightarrow 0$ as $x\rightarrow\pm\infty$. These fields correspond to a pulse of radiation moving in the +x direction. Maxwell equations place severe restrictions on the components. Show that we must have $F_1=G_1=0$, $G_3=F_2$, and $G_2=-F_3$, so that there are only two independent polarizations. Suppose $F_2(\xi)=G_3(\xi)=E_0\mathrm{Exp}(-\xi^2/a^2)$ and the other components are 0. (ξ stands for x-ct.) Make a sketch that shows a snapshot of the fields in space at time t.
5. Show that the function f = C Cos[kz] Cos[kct] is a solution of the wave equation. Determine the functions f_1 and f_2 (see (47.2)) for this solution. Explain what is meant by the statement that a standing wave is the superposition of traveling waves in opposite directions.

Section 48 Landau & Lifshitz Problem 1.

1. Show that the d'Alembertian operator $\Box \equiv -\partial^2/\partial x_i\partial x^i = \Delta - \partial^2/c^2\partial t^2$. Show that if $\mathbf{A} = \mathbf{A}_0\,\mathrm{Exp}[-i\,k^i\,x_i]$ in the four-dimensional wave equation that $k^ik_i=0$ (48.14).
2. Show (48.15) $T^{ik} = Wc^2\,k^ik^k/\omega^2$.
3. Calculate $\nabla\bullet\mathbf{S}$ and $\partial W/\partial t$ for a linearly polarized plane wave propagating in the z direction and polarized in the x direction, where **S** is the Poynting vector and W is the energy density. Explain the significance of the relationship between these two results for a given volume in space. What is the relationship between the electric field and magnetic field energy densities?
4. Polarized light is incident on a perfect polarizer, and it is observed that 20% of the light intensity gets through. What is the angle between the polarizer axis and the polarization direction of the light?
5. A polarized plane electromagnetic wave moves in the y direction, with the electric field in the ±x direction. What is the direction of the magnetic field at a point where the electric field is in the – x direction?
6. Consider a superposition of waves traveling in the z direction with fields $\mathbf{E}=\mathrm{Re}[\{E_1,E_2,0\}e^{i(kz-\omega t)}]$ and $\mathbf{H}=\mathrm{Re}[\{-E_2,E_1,0\}\,e^{i(kz-\omega t)}]$, where $E_1 = C_1\,e^{i\phi 1}$ and $E_2 = C_2\,e^{i\phi 2}$ with C_1 and C_2 real. Calculate the time-averaged energy flux S_{avg}. Suppose $E_1=C$ and $E_2=iC$. Describe in words and pictures the direction of **E** as a function of time, at a point on the xy plane. Describe in words and pictures the direction **E** as a function of z for a snapshot of the field at t=0.
7. The Lyman α spectral line emitted from hydrogen in a distant quasar is observed on earth to have a wavelength of 790 nm (near IR). This spectral line in terrestrial hydrogen is 122 nm (UV). How fast is the quasar receding from earth?
8. Two plane waves have the same frequency, wave vector and amplitude A, but opposite circular polarization. What are the amplitude and polarization of their superposition?
9. "Is there methane on Mars?" Kevin Zahnle, Richard S. Freedman, David C. Catling, Icarus 212 (2011) 493–503. "The martian lines are displaced from the core of the corresponding terrestrial lines by exploiting the Doppler shift when Mars is approaching or receding from Earth. Relative velocities can exceed 17 km/s. The Doppler shift for a relative velocity of 17 km/s is 0.17 cm^{-1} at 3000 cm^{-1}, which is enough to separate the centers of the martian and telluric lines, but not

enough to remove the martian lines from the wings of the much broader terrestrial lines." Figure: Transmission through Earth's atmosphere in the vicinity of the ν3 P4 methane lines. The wings of the P4 band are notably smooth. This makes the highest and lowest frequency lines of the four methane P4 lines relatively detectable when observed in blueshift or redshift, respectively. Doppler shifts of 0.16 cm^{-1} are assumed here for the illustration. Verify the expected shifts for 17 km/s relative velocity.



10. See the 1995 paper by Mayor & Queloz, which won them the 2019 Nobel Prize in Physics. (a) Sketch the orbit of a planet and the wobble of its star about their CM and how this would appear to an observer. (b) Write the exact expression for the frequency of light coming from the star as a function of time in terms of the orbital period T and the speed V of the instantaneous inertial frame attached to the star. (c) Since V<<c, write the expression for the frequency in this limit and the change in frequency $\Delta\omega$ about the proper frequency ω_0. (d) What resolving power $\omega_0/\Delta\omega$ would a spectrometer need to detect the motion if V = 13 m/s? (e) Look up the solar spectrum of the sun. How wide are the Balmer lines? What resolving power $\lambda_0/\Delta\lambda$ is needed to resolve these? How much smaller is the Doppler shift you calculated than is the Balmer linewidth? Hats off to Mayor & Queloz!

Section 49

1. Show that if f = Sum[f_n exp[- i ω_0 n t], {n, - ∞, ∞}] (49.1), then f_n = (1/T) Integral[f(t) exp [i n ω_0 t], {t, -T/2, T/2}] (49.2).
2. Show that the time average of purely periodic field $\langle f\rangle_t = f_0 = 0$, starting from the expansion of this field (49.1).
3. Show that for fields expandable in a continuous sequence of different frequencies (49.5) that the amplitude f_ω of each contribution has the form (49.6). Show that $f_{-\omega} = f_\omega^*$.
4. The figure shows actual experimental data for the periodic emission intensity of a far-infrared p-Ge laser. (Oscillations correspond to harmonics of the cavity round trip time, not the much faster THz frequency of the fields themselves.) What are

the fundamental period T and frequency ν_0 of the oscillations? For the upper trace, estimate the relative amplitudes f_n of all harmonics.

IEEE JOURNAL OF QUANTUM ELECTRONICS, VOL. 37, NO. 12, DECEMBER 2001

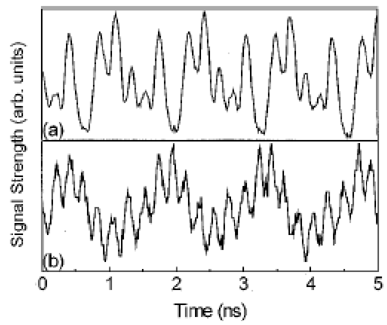


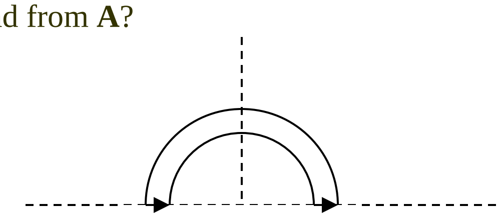
Fig. 3. Transient recording of 50.240-mm p-Ge crystal laser emission: (a) without and (b) with a Si spacer in the cavity.

Section 50

1. Show that if the polarization tensor is related to the field components according to (50.5), then its determinant vanishes.
2. For natural light show that the polarization tensor has the form $\rho_{\alpha\beta} = (1/2)\,\delta_{\alpha\beta}$. What is the determinant?
3. Show that for a circular polarized wave, the symmetric part of the polarization tensor is $(1/2)\delta_{\alpha\beta}$ while the antisymmetric part is $-(i/2)e_{\alpha\beta}A$ with $A = \pm 1$.
4. Show that the principal values of the symmetric part of the polarization tensor sum to one, i.e. $\lambda_1+\lambda_2=1$.
5. Show that if the coordinates are rotated about x by θ, so that y and z align with the principal axes of $S_{\alpha\beta}$, that $S'_{\alpha\beta}$ in the new coordinate system is diagonal.
6. Derive (50.11).
7. We measure Stokes parameter ξ_1 using a linear polarizer oriented first at +45 deg to the y (vertical) axis and then at -45 deg, measuring the transmitted intensities J_{45} and J_{-45}. Show $\xi_1 = (J_{45} - J_{-45})/J = 2\mathrm{Re}(\rho_{yz}) = [\langle E_yE_z^*\rangle_t + E_zE_y^*\rangle_t]/J$. Using the latter expression, find ξ_1 for (a) linear polarization along +45 deg; (b) linear polarization along -45 deg; (c) linear polarization along 0 deg; (d) Circular polarization (R or L)?
8. We measure Stokes parameter ξ_2 using a quarter wave plate (RHC light $E_y = iE_z$ goes to linear polarization at -45 deg $E_y = -E_z$) followed by an analyzer (linear polarizer) oriented first at -45 deg to the y (vertical) axis and then at +45 deg, measuring the transmitted intensities J_{-45} and J_{+45}. Show $\xi_2 = (J_{RHC} - J_{LHC})/J = 2\mathrm{Im}(\rho_{yz}) = -i[\langle E_yE_z^*\rangle_t - \langle E_zE_y^*\rangle_t]/J$. Using the latter expression, find ξ_2 for (a) circular polarization (R or L); (b) linear polarization along 0 deg; (c) linear polarization along +45 deg?
9. We measure Stokes parameter ξ_3 using a linear polarizer oriented first at +0 deg to the y (vertical) axis and then at 90 deg, measuring the transmitted intensities J_0 and J_{90}. Show $\xi_3 = (J_0 - J_{90})/J = (\rho_{yy} - \rho_{zz}) = [\langle E_yE_y^*\rangle_t - \langle E_zE_z^*\rangle_t]/J$. Using the latter expression, find ξ_3 for (a) linear polarization along 0 deg; (b) linear polarization along 90 deg; (c) linear polarization along 45 deg; (d) Circular polarization (R or L)?

Section 62

1. Show with $\phi = \chi(R/t)/R$ that the homogeneous d'Alembertian equation for ϕ reduces to $\partial^2\chi/\partial R^2 - (1/c^2)\, \partial^2\chi/\partial t^2 = 0$.
2. Suppose that at t=0 a current I is suddenly established in an infinite wire that lies on the z axis. What are the resulting electric and magnetic fields? Show that after a long time t>>r/c, the magnetic field is the same as the static field of a long wire with constant current I. What is the electric field for t>>r/c?
3. Confirm that the retarded potentials satisfy the Lorentz gauge conditions. First show that $\nabla\bullet(\mathbf{j}/R) = (1/R)\, \nabla\bullet\mathbf{j} + (1/R)\, \nabla'\bullet\mathbf{j} - \nabla'\bullet (\mathbf{j}/R)$, where $\mathbf{R} = \mathbf{r}-\mathbf{r}'$, ∇ denotes derivatives with respect to r, and ∇' denotes derivatives with respect to r'. Next, noting that $\mathbf{j}$(r', t-R/c) depends on r' both explicitly and through R, whereas it depends on r only through R, confirm that $\nabla\bullet\mathbf{j} = -(\partial\mathbf{j}/c\partial t) \bullet (\nabla R)$ and $\nabla'\bullet\mathbf{j} = -(\partial\rho/\partial t) - (\partial\mathbf{j}/c\partial t) \bullet (\nabla' R)$. Use this to calculate the divergence of **A** according to (62.10).
4. Suppose an infinite straight wire carries a linearly increasing current I(t) = k t, for t > 0. Find the E and H fields generated.
5. Suppose an infinite straight wire carries a sudden burst of current $I(t) = q_0\, \delta(t)$. Find the **E** and **H** fields generated.
6. A piece of wire bent into a loop, as shown in the figure with inner radius *a* and outer radius *b*, carries a current that increases linearly with time I(t) = k t. Calculate the retarded vector potential **A** and **E**-field at the origin. Can the magnetic field be found from **A**?



Section 63

1. Derive the Lienard-Wiechert potentials, i.e. the retarded potentials of a point charge, (63.5) from the four dimensional expression (63.3). Show that (63.2) results when **v** = 0. Show that $R_k R^k = 0$ where $R^k = [c(t - t'), \mathbf{r} - \mathbf{r}_0(t')]$.
2. Calculate the Poynting vector and energy density of the electromagnetic filed of a charged particle moving with constant velocity. Show that the field energy is carried along with the particle.
3. Find the dH field of a charge de moving uniformly with velocity v. Letting v de = I dx, where I is the current in a long straight wire, integrate dH over the length of the wire. Note that the result agrees with that obtained from Ampere's law even for v/c→1.
4. Suppose the acceleration of a fast particle is in the same direction as its velocity. Show that the radiation is zero along the direction of motion.
5. Find the expressions for the fields of an accelerated charge when its velocity is small.

Section 66

1. Verify the formula **H** = (1/c) (d**A**/dt) x **n** (66.3) for the magnetic field of a plane wave by direct computation of the curl of (66.2), dropping terms in $1/R_0^2$ in comparison with terms $\sim 1/R_0$.

Section 67 Landau & Lifshitz Problem 1

1. For the dipole **d**(t')=d_0 cosωt' $\mathbf{e}_z$, find the asymptotic vector potential in the wave zone.
2. One half of the intensity of electric dipole radiation is emitted in the angular range $(\pi/2)-(\alpha/2) \leq \theta \leq (\pi/2)+(\alpha/2)$. Determine α, which is called the half-intensity angle. Hint: You should end up with a cubic equation, which may be solved numerically, graphically, or by successive guessing.
3. Suppose a spherically symmetric charge distribution is oscillating purely in the radial direction, so that it remains spherically symmetric at every instant. How much radiation is emitted?
4. Suppose an electric dipole **d** rotates with a constant angular velocity ω about an axis perpendicular to the dipole moment. Find the radiation field and the Poynting vector by treating the rotating dipole as the superposition of two sinusoidally varying dipoles at right angles to each other.
5. The classical model of the hydrogen atom has the electron orbiting in a circle of radius r and with kinetic energy $E_k=(1/2)\ e^2/4\pi\varepsilon_0 r$, in Joules. Calculate the fractional energy radiated per revolution, PT/E_k, where T is the orbital period. Quantum mechanics prescribes that in the nth level v/c = 1/137n. Evaluate PT/E_k for n=2.
6. Consider an electron in a circular orbit of radius r about a proton. (a) Show that the total energy of the electron is $E = -(1/2)\ e^2/4\pi\varepsilon_0 r$, i.e. the negative of the kinetic energy, in S.I. units. (b) Assuming the orbit remains approximately circular, estimate the time for the electron to fall from R = 10 Å to R = 1 Å, assuming classical radiation of electromagnetic energy.
7. (a) Show that for a classical hydrogen atom, the time for the electron to spiral to the center is $T = (R/4c)[mc^2/(e^2/R)]^2$ in Gaussian units. (b) Find the life time of the classical H atom. The binding energy is 13.6 eV, the initial orbit radius is 0.53 Å, and the electron rest energy is 511 keV.
8. Find the spectrum of dipole radiation (67.10) in terms of wavelength.
9. Suppose a spherically symmetric shell of total charge Q is oscillating purely in the radial direction, so that it remains spherically symmetric at every instant. The radius of the shell can be described by the equation $r = r_0 \cos[t]$. Determine the mono-, di-, and quadru-pole moments of the distribution. What is the total radiation emitted?