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LXV. *On Transient Electric Currents.* *By* Prof. William Thomson*.

THE object of this communication is to determine the motion of electricity at any instant after an electrified conductor, of given capacity, charged initially with a given quantity of electricity, is put in connexion with the earth by means of a wire or other linear conductor of given form and resisting power. This linear conductor, which, to distinguish it from the other or principal conductor, will be called the discharger, is supposed to be of such small electrical capacity that the whole quantity of free electricity in it at any instant during the discharge is excessively small compared with the original charge of the principal conductor. Now any difference that can exist in the strength of the current at any instant in different parts of the discharger must produce accumulations of free electricity in the discharger itself, and therefore must be very small compared with the actual strength of the current depending on the discharge of the principal conductor. The strength of the current throughout the discharger will therefore be considered as the same at each instant, and, being measured by the quantity of electricity discharged per second, will be denoted by γ. Again, the conducting property and extent of surface of the principal conductor, and the resistance of the discharger, will be considered as so related that the potential throughout the principal conductor is uniform at each instant. Hence if q denote the quantity of electricity which the principal conductor possesses at any time t,

* Communicated by the Author, having been read at a meeting of the Glasgow Philosophical Society on the 19th of January, 1853.

we have

$$\gamma = -\frac{dq}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1).$$

Now, if C denote the electrical capacity of the principal conductor, that is, the quantity of electricity which it takes to make the potential within it unity, the mechanical value or the "potential energy" of the distribution of a quantity q upon it is $\frac{1}{2}\frac{q^2}{C}$. As this diminishes from the commencement of the discharge, and varies during the whole period of the discharge, corresponding mechanical effects must be produced in the discharger according to the general law of "*vis viva*," or of the preservation of mechanical energy. The mechanical effects in the discharger are of two kinds,—first, the excitation or alteration of electrical motion; secondly, the generation of heat. To estimate the first of these, it is necessary to know the mechanical value or the "actual energy" of an electrical current of given strength established and left without electromotive force in the discharger. In investigations which I have made towards a mechanical theory of electro-magnetic induction, I have found that the mechanical value of a current in a closed linear conductor is equal to the quantity of work that would have to be spent against the mutual electro-magnetic forces between its parts in bending it from its actual shape into any other shape, while a current of constant strength is sustained in it by an external electromotive force, together with the mechanical value of the current in the conductor thus altered. According to Faraday's experiments (Experimental Researches, § 1090, &c.), it appears that the actual energy of a current in a linear conductor doubled upon itself throughout its whole extent, is either nothing, or such as to produce no sensible spark when the circuit is suddenly opened at any point; that is, that what can be obviously interpreted as inertia of electricity either does not exist, or produces but insensible effects compared to those which have been attributed to the "induction of a current upon itself." According to these views, the actual energy of an electric current of given strength in a given closed linear conductor would be determined analytically by calculating the amount of work against mutual electro-magnetic actions required to double it upon itself throughout its whole extent; but it may be that a more complete knowledge of the circumstances will show a term depending on electrical inertia which must be added to the quantity determined in that way to give the entire mechanical value of the current. However this may be, and whether the linear conductor be open or closed, it is obvious that the actual energy of a current established in it and left without electromotive force must be proportional to the square of the strength of the current, and this is all that is required to be known for the present investigation. Let then $\frac{1}{2}A\gamma^2$ denote the actual energy of a current of strength, γ, in the linear conductor which serves for discharger in the arrangement which forms the subject of the present investigation, A being a constant which may be called the electrodynamic capacity of the discharger. The work spent in exciting electrical motion during the time dt will be

$$d\left(\frac{1}{2}A\gamma^2\right).$$

Again, the work done in generating heat in the same time is, according to Joule's law,

$$k\gamma^2 dt,$$

if k denote the "galvanic resistance" of the discharger, or the mechanical equivalent of the heat generated in it, in the unit of time by a current of unit strength*. Now the loss of potential energy from the principal conductor, in the time dt, being $-d\left(\frac{1}{2}\frac{q^2}{C}\right)$, is entirely spent in producing these effects; and therefore

$$-d\left(\frac{1}{2}\frac{q^2}{C}\right)=d\left(\frac{1}{2}A\gamma^2\right)+k\gamma^2 dt \quad . \quad . \quad . \quad . \quad (2).$$

This equation and (1), with the conditions

$$q=Q, \text{ and } \gamma=0, \text{ when } t=0 \quad . \quad . \quad . \quad . \quad (3),$$

are sufficient for the determination of q and γ for any value of t, that is, for the complete solution of the problem.

By (1) we have

$$-d\left(\frac{1}{2}\frac{q^2}{C}\right)=\frac{q}{C}\gamma dt,$$

and (2) becomes

$$\frac{q}{C}\gamma dt=A\gamma d\gamma+k\gamma^2 dt,$$

from which we find

$$q=C\left(A\frac{d\gamma}{dt}+k\gamma\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (4).$$

Substituting for γ its value by (1), we obtain

$$\frac{d^2q}{dt^2}+\frac{k}{A}\frac{dq}{dt}+\frac{1}{CA}q=0 \quad . \quad . \quad . \quad . \quad . \quad (5).$$

The general solution of this equation is

$$q=K\epsilon^{\rho t}+K'\epsilon^{\rho' t},$$

* See a paper entitled "Application of the Principle of Mechanical Effect to the Measurement of Electromotive Forces and Galvanic Resistances," Phil. Mag. Dec. 1851.

where ρ and ρ' are the roots of the equation

$$x^2+\frac{k}{\mathrm{A}}x+\frac{1}{\mathrm{CA}}=0.$$

Using equations (3) and (1) to determine the arbitrary constants K and K', and to derive an expression for γ, we obtain a complete solution of the problem which is expressed most conveniently by one or other of the following sets of formulæ, according as ρ and ρ' are real or imaginary:—

$$\left.\begin{aligned} q&=\frac{\mathrm{Q}}{2\alpha\mathrm{A}}\epsilon^{-\frac{k}{2\mathrm{A}}t}\left\{\left(\alpha\mathrm{A}+\frac{k}{2}\right)\epsilon^{\alpha t}+\left(\alpha\mathrm{A}-\frac{k}{2}\right)\epsilon^{-\alpha t}\right\}\\ \gamma&=\frac{\mathrm{Q}}{2\alpha\mathrm{AC}}\epsilon^{-\frac{k}{2\mathrm{A}}t}\{\epsilon^{\alpha t}-\epsilon^{-\alpha t}\}\\ &\text{where}\\ \alpha&=\left(\frac{k^2}{4\mathrm{A}^2}-\frac{1}{\mathrm{CA}}\right)^{\frac{1}{2}}; \end{aligned}\right\} \quad . \quad (6),$$

$$\left.\begin{aligned} q&=\frac{\mathrm{Q}}{\alpha'\mathrm{A}}\epsilon^{-\frac{k}{2\mathrm{A}}t}\left\{\alpha'\mathrm{A}\cos(\alpha' t)+\frac{k}{2}\sin(\alpha' t)\right\}\\ \gamma&=\frac{\mathrm{Q}}{\alpha'\mathrm{AC}}\epsilon^{-\frac{k}{2\mathrm{A}}t}.\sin(\alpha' t)\\ &\text{where}\\ \alpha'&=\left(\frac{1}{\mathrm{CA}}-\frac{k^2}{4\mathrm{A}^2}\right)^{\frac{1}{2}}. \end{aligned}\right\} \quad . \quad . \quad (7).$$

Among numerous other beautiful applications of his "electro-dynamometer," Weber has shown a method of determining what he calls the "duration"* of a transient electric current. In accordance with the terms he uses, the duration, and the *mean strength* of a transient current may be defined respectively as the duration and the strength that a uniform current must have to produce the same effects on the electro-dynamometer and on an ordinary galvanometer; so that if T and Γ denote the duration and the mean strength of a current, of which the actual strength at any instant is γ, we have

$$\left.\begin{aligned} \mathrm{T}&=\frac{\left\{\int_0^\infty \gamma dt\right\}^2}{\int_0^\infty \gamma^2 dt}\\ \Gamma&=\frac{\int_0^\infty \gamma^2 dt}{\int_0^\infty \gamma dt} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (8);$$

* "Bestimmung der Dauer momentaner Ströme mit dem Dynamometer nebst Anwendung auf physiologische Versuche," § 13 of Weber's *Electrodynamische Maasbestimmungen*, Leipsic 1846.

since the electro-dynamometer indicates the value of $\int_0^\infty \gamma^2 dt$, and the ordinary galvanometer that of $\int_0^\infty \gamma dt$. If for γ we use the expression in either (6) or (7), we find

$$\int_0^\infty \gamma^2 dt = \frac{1}{2}\frac{Q^2}{kC}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (9),$$

as might have been foreseen, independently of the complete solution, by considering that, as the heat generated in the discharger is the sole final effect produced by the discharge, the mechanical value of the whole heat generated, or $\int_0^\infty k\gamma^2 dt$, must be equal $\frac{1}{2}\frac{Q^2}{C}$, the mechanical value of the primitive charge, and that k has been assumed to have a constant value during the discharge. Again, we derive from (1) and (3),

$$\int_0^\infty \gamma dt = Q \quad . \quad . \quad . \quad . \quad . \quad . \quad (10).$$

Hence in the present case the expressions for the duration and mean strength of the current become

$$\left.\begin{aligned} T &= 2kC \\ \Gamma &= \frac{Q}{2kC} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (11).$$

We conclude that the "duration" of the discharge is proportional to the capacity of the principal conductor, and to the resistance of the discharger; and that it is independent of the quantity of electricity in the primitive charge, and of the electro-dynamical capacity (denoted above by A) of the discharger. The only doubtful assumption involved in the preceding investigation is that of the constancy of k during the discharge. Joule's experiments show that the value of k remains unchanged for the same metallic conductor kept at the same temperature, whatever be the strength of the current passing through it; but that it would be increased by any elevation of temperature, whether produced by the current itself or by any other source of heat, since an elevation of temperature always increases the galvanic resistance of a metal. When large quantities of electricity are discharged, or when the discharger is a very fine wire, great augmentations and diminutions may therefore take place in the value of k, and therefore the solution obtained above is not applicable to such cases. If, however, k denote the mean resistance of the discharger during the discharge, that is, a quantity such that

$$k\int_0^\infty \gamma^2 dt = \int_0^\infty \kappa\gamma^2 dt \quad . \quad . \quad . \quad . \quad . \quad (12),$$

where κ denotes the actual resistance at any instant of the discharge, the last equations (11) become merely the expressions for the elements determined by Weber from observations by means of the two instruments, and they are therefore applicable to all cases.

In the experiments described by Weber, the discharger consisted of a wet cord of various lengths, and all the wire of the electro-dynamometer and the ordinary galvanometer. The "durations" of the discharge in different cases were found to be nearly proportional to the length of the wet cord, and equal to ·0851 sec., or about $\frac{1}{12}$th of a second, when the length of the cord was 2 metres. As the principal resistance must undoubtedly have been in the wet cord, we may infer from equations (11) and (12) that the mean resistances in all the different discharges must have been nearly proportional to its lengths. In some of the experiments the length was only $\frac{1}{4}$ of a metre, and the value of T was about ·0095 of a second. Hence the current in the string must have been about eight times as intense as when the duration was $\frac{1}{12}$, since the quantities of electricity discharged were nearly equal in the different cases. We conclude that the intensity of the current cannot have materially affected the resisting power of the cord; probably not nearly so much as inevitable differences arising from accidental circumstances in the different experiments. Hence, although nothing is known with certainty regarding the non-electrolytic resistance of liquid conductors in general, it is probable that the whole resistance of the discharger in Weber's experiments must have been nearly independent of the strength of the current at each instant; and we may therefore consider the general solution expressed above by (6) or (7) as at least approximately applicable to these cases.

The two forms (6) and (7) of the solution of the general problem indicate two kinds of discharge presenting very remarkable distinguishing characteristics. Thus in all cases in which $\frac{k^2}{4A^2}$ exceeds $\frac{1}{CA}$, the exponentials in (6) are all real; and the solution expressed by these equations shows that the quantity of electricity on the principal conductor diminishes continuously, and that the discharging current commences and gradually increases in strength up to a time given by the equation

$$-\left(\frac{k}{2A}-\alpha\right)\epsilon^{-\left(\frac{k}{2A}-\alpha\right)t}+\left(\frac{k}{2A}+\alpha\right)\epsilon^{-\left(\frac{k}{2A}+\alpha\right)t}=0,$$

or

$$t=\frac{1}{2\left(\frac{k^2}{4A^2}-\frac{1}{CA}\right)^{\frac{1}{2}}}\log\frac{\frac{k}{2A}+\left(\frac{k^2}{4A^2}-\frac{1}{CA}\right)^{\frac{1}{2}}}{\frac{k}{2A}-\left(\frac{k^2}{4A^2}-\frac{1}{CA}\right)^{\frac{1}{2}}}\quad . \quad . \quad (13);$$

after which it diminishes gradually, and, as well as the quantity of electricity on the principal conductor, becomes nothing when $t=\infty$. On the other hand, when $\frac{1}{\text{CA}}$ exceeds $\frac{k^2}{4\text{A}^2}$, the exponentials and trigonometrical functions in (7) are all real; and the solution expressed by these equations shows that the principal conductor loses its charge, becomes charged with a less quantity of the contrary kind of electricity, becomes again discharged, and after that charged with a still less quantity of the same kind of electricity as at first, and so on for an infinite number of times before equilibrium is established. The times at which the charge of either kind of electricity on the principal conductor is a maximum, being those at which γ vanishes, are the roots of the equation $\sin(\alpha' t)=0$, and therefore follow successively from the commencement at equal intervals $\frac{\pi}{\alpha'}$. The quantities constituting the successive maximum charges are

$$\text{Q},\quad -\text{Q}\epsilon^{-\frac{k\pi}{2\text{A}\alpha'}},\quad \text{Q}\epsilon^{-\frac{2k\pi}{2\text{A}\alpha'}},\quad -\text{Q}\epsilon^{-\frac{3k\pi}{2\text{A}\alpha'}},\ \&\text{c.}\quad .\ (14);$$

each being less in absolute magnitude than that which precedes it in the ratio of $1:\epsilon^{\frac{k\pi}{2\text{A}\alpha'}}$, and of the opposite kind. The strength of current will be a maximum in either direction when $\frac{d\gamma}{dt}=0$, or when

$$\frac{k}{2\text{A}}\sin(\alpha' t)=\alpha'\cos(\alpha' t);$$

and therefore if T_1, T_2, &c. denote the successive times when this is the case, measured from the commencement of the discharge, and γ_1, γ_2, &c. the corresponding maximum values of the strength of the current, and if θ denote the acute angle satisfying the equation $\tan\theta=\frac{2\text{A}\alpha'}{k}$, we have

$$\text{T}_1=\frac{\theta}{\left(\frac{1}{\text{CA}}-\frac{k^2}{4\text{A}^2}\right)^{\frac{1}{2}}},\quad \text{T}_2=\frac{\theta+\pi}{\left(\frac{1}{\text{CA}}-\frac{k^2}{4\text{A}^2}\right)^{\frac{1}{2}}},\quad \text{T}_3=\frac{\theta+2\pi}{\left(\frac{1}{\text{CA}}-\frac{k^2}{4\text{A}^2}\right)^{\frac{1}{2}}},\ \&\text{c}$$

$$\gamma_1=\frac{\text{Q}}{\text{A}\left(\frac{1}{\text{CA}}-\frac{k^2}{4\text{A}^2}\right)^{\frac{1}{2}}}\epsilon^{-\frac{k}{2\text{A}}\text{T}_1},\quad \gamma_i=\left(\frac{-1}{\epsilon^{\frac{k\pi}{2\text{A}\alpha'}}}\right)^i\gamma_1.$$

It is probable that many remarkable phænomena which have been observed in connexion with electrical discharges are due to the oscillatory character which we have thus found to be possessed when the condition

$$\frac{1}{CA} > \frac{k^2}{4A^2} \text{ or } C < \frac{4A}{k^2} \quad \ldots \ldots \quad (15)$$

is fulfilled. Thus if the interval of time $\frac{\pi}{\left(\frac{1}{CA} - \frac{k^2}{4A^2}\right)^{\frac{1}{2}}}$, at which the successive instants when the strength of the current is a maximum follow one another, be sufficiently great, and if the evolution of heat in any part of the circuit by the current during several of its alternations in direction be sufficiently intense to produce visible light, a succession of flashes diminishing in intensity and following one another rapidly at equal intervals will be seen. It appears to me not improbable that double, triple, and quadruple flashes of lightning which I have frequently seen on the continent of Europe, and sometimes, though not so frequently in this country, lasting generally long enough to allow an observer, after his attention is drawn by the first light of the flash, to turn his head round and see distinctly the course of the lightning in the sky, result from the discharge possessing this oscillatory character. A corresponding phænomenon might probably be produced artificially on a small scale by discharging a Leyden phial or other conductor across a very small space of air, and through a linear conductor of large electrodynamic capacity and small resistance. Should it be impossible on account of the too great rapidity of the successive flashes for the unaided eye to distinguish them, Wheatstone's method of a revolving mirror might be employed, and might show the spark as several points or short lines of light separated by dark intervals, instead of a single point of light, or of an unbroken line of light, as it would be if the discharge were instantaneous, or were continuous and of appreciable duration.

The experiments by Riess and others on the magnetization of fine steel needles by the discharge of electrified conductors, illustrate in a very remarkable manner the oscillatory character of the discharge in certain circumstances; not only when, as in the case with which we are at present occupied, the whole mechanical effect of the discharge is produced within a single linear conductor, but when induced currents in secondary conductors generate a portion of the final thermal equivalent.

The decomposition of water by electricity from an ordinary electrical machine, in which, as has been shown by Faraday, more than the electro-chemical equivalent of the whole electricity that passes appears in oxygen and hydrogen rising mixed from each pole, is probably due to electrical oscillations in the discharger consequent on the successive sparks*. Thus, if the general law of electro-chemical decomposition be applicable to currents of such very short duration as that of each alternation in such an oscillatory discharge as may take place in these circumstances, there will be decomposed altogether as much water as is electro-chemically equivalent to the sum of the quantities of electricity that pass in all the successive currents in the two directions, while the quantities of oxygen and hydrogen which appear at the two electrodes will differ by the quantities arising from the decomposition of a quantity of water electro-chemically equivalent to only the quantity of electricity initially contained by the principal conductor. The formulæ investigated above will be applicable to this case if the end of the discharging train next the machine be placed in metallic communication with an insulated conductor, satisfying the conditions laid down with reference to the "principal conductor" at the commencement of this paper, and if this conductor be successively electrified by sparks from the machine. The whole quantity of water decomposed will therefore be the electro-chemical equivalent of the sum of the absolute values of the quantities of electricity flowing out of and into the principal conductor during the successive alternations of the current, that is, according to the preceding formulæ, the electro-chemical equivalent of the quantity,

$$Q(1+2\epsilon^{-\frac{k\pi}{2A\alpha'}}+2\epsilon^{-\frac{2k\pi}{2A\alpha'}}+\&c.), \text{ or } Q\frac{1+\epsilon^{-\frac{k\pi}{2A\alpha'}}}{1-\epsilon^{-\frac{k\pi}{2A\alpha'}}},$$

of electricity. This quantity will be the greater the more nearly $\epsilon^{-\frac{k\pi}{2A\alpha'}}$ approaches to unity, that is the greater is $\frac{2A\alpha'}{k}$ or $\left(\frac{4A}{kC}-1\right)^{\frac{1}{2}}$, or the greater is $\frac{4A}{kC}$. Hence the greater the electro-dynamic capacity of the discharger, the less its resistance, and the less

* This explanation occurred to me about a year and a half ago, in consequence of the conclusions regarding the oscillatory nature of the discharge in certain circumstances drawn from the mathematical investigation. I afterwards found that it had been suggested, as a conjecture by Helmholtz, in his *Erhaltung der Kraft* (Berlin 1847), in the following terms :—

"** It is easy to explain this law if we assume that the discharge of a battery is not a simple motion of the electricity in one direction, but a backward and forward motion between the coatings, in oscillations which become continually smaller until the entire *vis viva* is destroyed by the sum of the resistances. The notion that the current of discharge consists of alternately opposed currents is favoured by the alternately opposed magnetic actions of the same; and secondly, by the phænomena observed by Wollaston while attempting to decompose water by electric shocks, that both descriptions of gases are exhibited at both electrodes." [Quoted from the translation in Taylor's New Scientific Memoirs, Part II.]

the electro-statical capacity of the principal conductor, the greater will be the whole quantity of water decomposed. Probably the best arrangement in practice would be one in which merely a small ball or knob is substituted for a principal conductor fulfilling the conditions prescribed above; but those conditions not being fulfilled, the circumstances would not be exactly expressed by the formulæ of the present communication. The resistance would be much diminished, and consequently the whole quantity of water decomposed much increased, by substituting large platinum electrodes for the mere points used by Wollaston; but then the oxygen and hydrogen separated during the first direct current would adhere to the platinum plates, and would be in part neutralized by combination with the hydrogen and oxygen brought to the same plates respectively by the succeeding reverse current; and so on through all the alternations of the discharge. In fact, if the electrodes be too large, all the equivalent quantities of the two gases brought successively to the same electrode will recombine, and at the end of the discharge there will be only oxygen at the one electrode and only hydrogen at the other, in quantities electro-chemically equivalent to the initial charge of the principal conductor. Hence we see the necessity of using very minute electrodes, and of making a considerable quantity of electricity pass in each discharge, so that each successive alternation of the current may actually liberate from the electrodes some of the gases which it draws from the water. Probably the most effective arrangement would be one in which a Leyden phial or other body of considerable capacity is put in connexion with the machine and discharged in sparks through a powerful discharger, not only of great electro-dynamic capacity and of as little resistance as possible except where the metallic communication is broken in the electrolytic vessel, but of great electro-statical capacity also, so that all, or as great a portion as possible, of the oscillating electricity may remain in it and not give rise to successive sparks across the space of air separating the discharger from the source of the electricity.

The initial effect of a uniform electromotive force in establishing a current in a linear conductor may be determined by giving C and Q infinite values in the preceding formulæ, and $\frac{Q}{C}$ a finite value V, which will amount to supposing the potential at one end of the discharger to be kept constantly at the value V, while the potential at the other end is kept at zero. The formulæ suitable to this case, which is obviously a case of non-oscillatory discharge, are (6); and from them we deduce

$$\gamma = \frac{V}{k}\left(1 - \epsilon^{-\frac{k}{A}t}\right). \quad . \quad . \quad . \quad . \quad . \quad (16),$$

which agrees with conclusions arrived at by Helmholtz and others.

This result shows how, when a linear conductor, initially in a state of electrical equilibrium, becomes subjected to a constant electromotive force V between its extremities, a current commences in it and rises gradually in strength towards the limit $\frac{V}{k}$. This limit cannot be perfectly reached in any finite time, although in reality only a very minute time elapses from the commencement in ordinary cases, until the current acquires so nearly the full strength $\frac{V}{k}$ that no further augmentation is perceptible*.

The equations (6), expressing generally a continuous discharge, assume the following forms when A is infinitely small,

$$\left.\begin{aligned} \gamma &= \frac{Q}{Ck}\epsilon^{-\frac{t}{kC}} \\ q &= Q\epsilon^{-\frac{t}{kC}} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (17),$$

which show how, when anything like electrical inertia is insensible, the current commences instantly with its maximum strength, and then gradually sinks as the charge gradually and permanently leaves the principal conductor.

One of the results of the preceding investigation shows a very important application that may be made of Weber's experimental determination of the "duration" of a transient current, to enable us to determine the numerical relation between electro-statical and electro-magnetic units. For if σ denote the quantity of electricity in electro-statical measure which passes in the unit of time to constitute a current of unit strength in electro-magnetic measure, the strength of a current expressed above by γ will be $\frac{1}{\sigma}\gamma$, in terms of the electro-magnetic unit; and if K denote the resistance, in absolute electro-magnetic measure, of a linear conductor of which the resistance measured as above in terms of the electro-statical unit is k, we have

$$K\left(\frac{\gamma}{\sigma}\right)^2 = k\gamma^2,$$

which gives

$$k = \frac{K}{\sigma^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18).$$

* See a paper by Helmholtz in Poggendorff's *Annalen*, 1852, which contains valuable researches, both theoretical and experimental, on this subject.

Hence the first of equations (11) gives

$$\sigma = \left(\frac{2KC}{T}\right)^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (19).$$

Now Weber has not only determined T, in certain cases alluded to above, but has shown how K may be determined for any linear conductor. Again, the value of C for a Leyden phial is, according to Green and Faraday, expressed by the equation

$$C = I \frac{S}{4\pi\tau} \quad . \quad . \quad . \quad . \quad . \quad . \quad (20),$$

where S denotes the area of one side of the coated glass, τ the thickness of the glass, and I the specific inductive capacity of its substance*. Thus, either by using a Leyden phial or some other conductor of which the electro-statical capacity can be found, and by determining the "duration" of a discharge from it through a linear conductor, of which the resistance in absolute electro-magnetic measure has been determined, we have everything that is required for calculating the value of σ by means of equation (19). The determination of this quantity enables us to compare the electro-statical and electro-magnetic measures of electromotive force. For if V denote a constant difference of potentials kept up between the two extremities of a linear conductor, and if γ denote the strength of the uniform current that results, we have, according to the conclusions drawn above from (16),

$$\gamma = \frac{V}{k}.$$

But if F denote the electromotive force between the ends of the linear conductor in electro-magnetic measure, we have

$$\frac{1}{\sigma}\gamma = \frac{F}{K} = \frac{F}{\sigma^2 k},$$

and therefore

$$F = \sigma V \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21).$$

Many different ways of determining the value of this important element, σ, besides that suggested above, might probably be put in practice. Perhaps the most accurate would be to take a multiple galvanic battery of constant and known electromotive force (consisting, for instance, of a hundred or more cells of Daniell's), and measure the force of attraction between two plane conduct-

* The value of I for flint-glass is, according to Faraday (Experimental Researches, Series XI.), greater than 1·76, for shell-lac about 2, for sulphur rather more than 2·2. See a paper "On the Elementary Laws of Statical Electricity," § 8. (Cambridge and Dublin Mathematical Journal, vol. i. Nov. 1845.)

ors parallel to one another at a very small measured distance asunder, with only air between them. If X be the force of attraction thus measured, a the distance between the conductors, S the area of each or of that portion of each which is directly opposed to the other, and V the difference of the electrical potential kept up between them by the battery, we should have

$$V = a\left(\frac{8\pi X}{S}\right)^{\frac{1}{2}};$$

and therefore, if F be the electromotive force of the battery in electro-magnetic measure,

$$\sigma = \frac{F}{a\left(\frac{8\pi X}{S}\right)^{\frac{1}{2}}}.$$